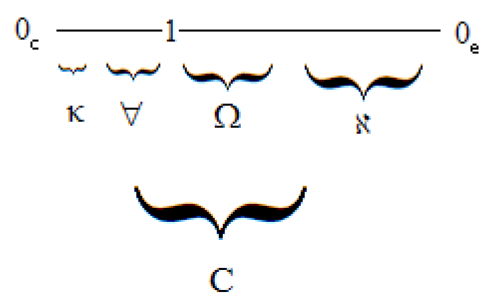
C3:

The Canonized Cardinal Continuum

$$\left.\begin{array}{l} \kappa\ \{\ \kappa_0 \to 0_c\ \} \\ \forall\ \{\ \kappa_0 \to 1\ \} \\ \Omega\ \{\ \aleph_0 \to 1\ \} \\ \aleph\ \{\ \aleph_0 \to 0_e\ \} \\ 0\ \{\ 0_c \to 0_e\ \} \end{array}\right\}\ C\ \{\{0\},\{\ 0_c \to 0_e\ \}\}$$

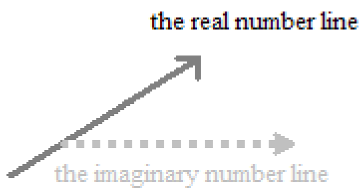


"The consistency of the incompleteness
completes the inconsistency of the continuum."

By Gavin Wince

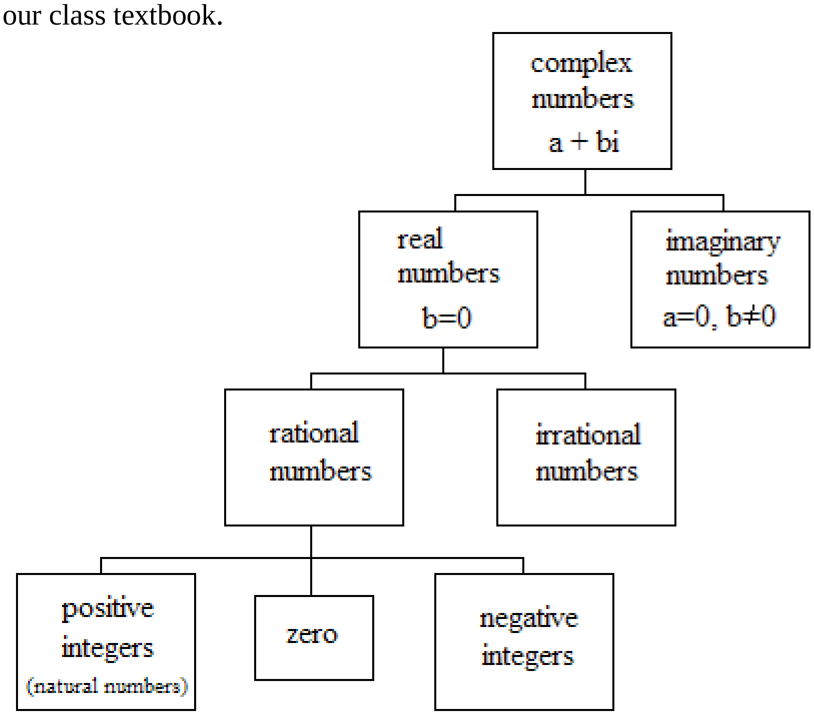
Introduction

During my sophomore year of High School, I had a mathematical experience that changed my life and perspective on reality. Grant Philips of Reo Mesa High School in Oxnard, California began to teach us about imaginary numbers in his Algebra class.



He drew a line across the chalkboard, called it the real number line, and then did something remarkable: he pointed the chalk on the chalkboard perpendicularly away from the real number line demonstrating the orientation of how the *imaginary* number line exists in relation to the real number line. Something inside me clicked as I fell in love with abstract mathematics at age 16.

After dropping out of High School during the fall of my junior year and admitting my self into the local Community College that spring, an experience in another math class mutated my deep interests into willful action. During a college Algebra class, the instructor began to explain the definitions of the various types of numbers discussing the subject in reference to figure 1-12 in our class textbook.



Gustafson & Frisk, College Algebra 4th Edition, Brooks/Cole, 1990, p.66, Chapter 1: Basic Concepts; Figure 1-12

It appeared that something was missing from the right-hand corner of this figure that, if included, would complete the picture so to speak. I began looking at mathematic as missing something that specifically had to do with what made imaginary numbers different from real, rational numbers different from irrational, and whatever it is that is *missing* being different from everything else.

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1.1- An Infinite Series with an Infinitesimal Final Term

Let there be a sequence with infinite terms;

$x_1 , x_2 , x_3 , \dots , x_n$

Each term of the sequence is the sum of all subsequent term(s) added to the next consecutive term as a series, where each term in the series is some product of $1/2^n$. The first term in x_1 corresponds to the first term of the series.

$x_1 = 1/2^1 = 1/2 = 0.5$

The 1st and 2nd terms in x_2 correspond to the first and second terms of the series.

$x_2 = 1/2^1 + 1/2^2 = 1/2 + 1/4 = 0.75$

Each term of the sequence $x_1 , x_2 , x_3 , \dots , x_n$ is a partial and finite sum such that each consecutive term of a series is half the value of the previous term and each term of the sequence is closer to 1.0 than the previous term of the sequence.

$x_1 = 1/2^1 = 1/2 = 0.5$

$x_2 = 1/2^1 + 1/2^2 = 1/2 + 1/4 = 0.75$

$x_3 = 1/2^1 + 1/2^2 + 1/2^3 = 1/2 + 1/4 + 1/8 = 0.875$

$x_4 = 1/2^1 + 1/2^2 + 1/2^3 + 1/2^4 = 1/2 + 1/4 + 1/8 + 1/16 = 0.9375$

etc.

$x_n = 1.0$

At the end or limit of the infinite sequence, the final term of the sequence is 1.0

$x_n = x_1 + (x_2 - x_1) + (x_3 - x_2) + \dots + (x_n - x_{n-1}) = 1.0$

$x_n = 1/2^1 + 1/2^2 + 1/2^3 + 1/2^4 + \dots + 1/2^n = 1.0$

$= 1/2 + 1/4 + 1/8 + 1/16 + \dots + 1/2^n = 1.0$

In this example we can see that as the number of finite sums of the sequence approaches the limit infinity, the last term of the sequence equals one.

$x_n = 1.0$

If we are going to assume that the last term of the sequence equals one, it can be deduced that, prior to the last term in the sequence, some finite sum in the series occurs where:

$$x_{n-1} = 0.999\ldots$$

$$x_{n-1} = 1/2^1 + 1/2^2 + 1/2^3 + 1/2^4 + \ldots + 1/2^{n-1} = 0.999\ldots$$

Therefore, at the limit, the last term of the series of the last term of the sequence would be the term, which, when added to the sum 0.999… equals 1.0

$$x_n = x_1 + (x_2 - x_1) + (x_3 - x_2) + \ldots + (x_n - x_{n-1}) = 1.0$$

$$= 1/2 + 1/4 + 1/8 + \ldots + 1/2^n = 1.0$$

$$= 0.999\ldots + 1/2^n = 1.0$$

$$= 0.999\ldots + \text{some infinitesimal} = 1.0$$

Assuming that a finite number raised to the power of infinity is an infinite quantity, it is deducible that the reciprocal is an infinitesimal quantity. Therefore, we will let the last term in the series ($1/2^n$ where n is infinite) equal some infinitesimal quantity.

$$1/2^n = (x_n - x_{n-1}) = (1.0 - 0.999\ldots) = \text{some infinitesimal.}$$

1.2- Degrees of Infinity

Take note that the inverse of the last term is 2^n.

$$[1/2^n]^{-1} = 2^n$$

Since 2^n is the power set of n we will now raise the question whether or not the power set of an infinite set is greater than or equal to the original set?

Either:

$$1/2^\infty = 1/\infty, \quad \text{and} \quad 2^\infty = \infty$$

or:

$$1/2^\infty < 1/\infty, \quad \text{and} \quad 2^\infty > \infty$$

Let there exist at least two degrees of infinity. The least degree of infinity is the set of all countable numbers, say the set of all natural numbers:

$$\{ 1, 2, 3, \ldots , n \}$$

The greater degree is the continuum, or set of all countable and uncountable numbers.

$$C\{ \circ\!\longrightarrow n\}$$

The set of all countable numbers, or natural numbers, is a subset of the continuum. Since the set of all natural numbers is a subset of the continuum, it is reasonable to assume that the set of all natural numbers is less in degree of infinity than the set containing the continuum.
However, it does not follow that the power set of the set of all natural numbers has the same degree of infinity as the continuum, since this would leave only two degrees of infinity. It might be more intuitive to assume that there are infinite degrees of infinity, as there are infinite finite numbers, than to presume a finite degree of infiniteness. Why would there be less degrees of infinity than number of finite numbers?

Let the set of all natural numbers be denoted as $\aleph_0$: the first infinite or cardinal number. Let the next hypothesized cardinal number be denoted as $\aleph_1$. We will entertain the possibility that $\aleph_1$ is a third degree of infiniteness and that there could be $\aleph_n$ degrees of infiniteness.

The power set of the set of all natural numbers has one of the following three solutions:

1). The power set of $\aleph_0$ equals the set of all natural numbers:

$$2^{\aleph_0} = \aleph_0$$

2). The power set of $\aleph_0$ equals the set of the continuum:

$$2^{\aleph_0} = C$$

3). The power set of $\aleph_0$ equals some intermediary infinite set that is greater than the set of natural numbers but is less than the set of the continuum:

$$2^{\aleph_0} = \aleph_1 \qquad \qquad \aleph_1 < C$$

To determine if the power set of the natural numbers is greater than or equal to the set of the natural numbers, or if the power set of the natural numbers is the set of the continuum, we will algebraically examine them in relation to one another.

Assuming that if:

$\aleph_1 \neq C$ and $\aleph_1 < C$

and that:

$$2^{\aleph_0} = \aleph_1$$

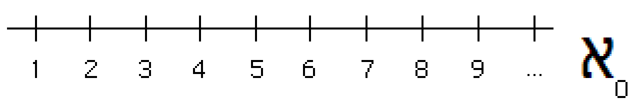
Either:

$\aleph_0 = \aleph_1 = \aleph_2 = \aleph_3 = \aleph_4 = \ldots = C$, and there really is only one degree of infinity,

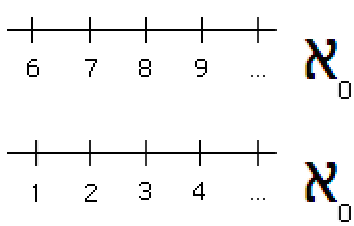
or:

$\aleph_0 < \aleph_1 < \aleph_2 < \aleph_3 < \aleph_4 < \ldots < C$, and there are infinite degrees of infinity, with the continuum being the greatest.

If we subtract (or add) a finite quantity with an infinite quantity, it can be shown that the infinite quantity is still the same quantity. For instance, imagine a line that is infinitely long. Assume it has a length the size of the set of all natural numbers.

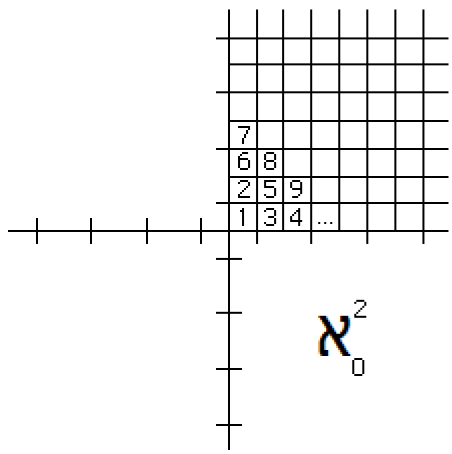


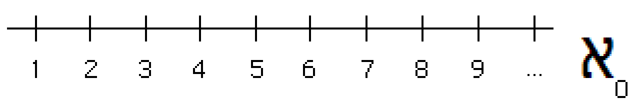
If we were to subtract from the left side of this line, say five units, our line would start with 6 and continue from there. We could still make a 1-1 correspondence between our new line and the original line, such that 6 corresponds to 1, 7 corresponds to 2, 8 corresponds to 3, 9 corresponds to 4, etc.



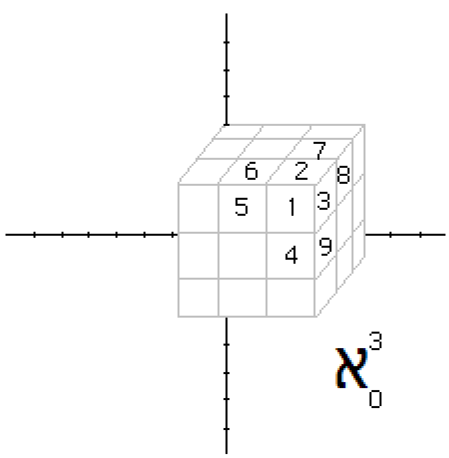
Therefore it can be shown that both lines contain the same infinite units of numbers, namely $\aleph_0$.

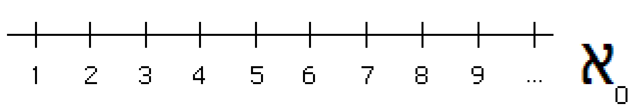
Also, raising the set of all natural numbers to the power of a finite number does not increase the number of members of that set. For example, if we square the line and turn it into a plane, we can see that though there is an added dimension through which quantities can arise, we can still make a 1-1 correspondence between the plane and the original line.





Again, if we cube the original line, we can still make a 1-1 correspondence between the three dimensions and the original line.





So, more or less, it can be argued that:

$$\aleph_0 = 1 \pm \aleph_0 = 2 \pm \aleph_0 = 3 \pm \aleph_0 = 4 \pm \aleph_0 = \ldots = n \pm \aleph_0$$

and,

$$\aleph_0 = \aleph_0^2 = \aleph_0^3 = \aleph_0^4 = \aleph_0^5 = \ldots = \aleph_0^n$$

(where n *is a finite quantity)*

However, the extreme difference between finite and infinite quantities is so vast that one could not expect that an infinite raised to the power of a finite could yield the same results as raising an infinite to an infinite power, such as:

$$\aleph_0^2 = \aleph_0 \quad = ? = \quad \aleph_0^{\aleph_0} = \aleph_0$$

Similarly, it does not seem intuitive that the power set of an infinite set, such as the set of all natural numbers, would have the same number of members as the original set when even finite power sets contain more members than the original set itself. It is inherent to the definition of what it is to be a power set that a power set contains more members than the original set. Therefore, it also does not intuitively deduce that:

$$2^{\aleph_0} = \aleph_0$$

1.3- Infinitesimals as reciprocals for Degrees of Infinity

A deeper examination of attempting to algebraically define the relationship between infinites and infinitesimals reveals something that serves as a resolution to the aforementioned issue at hand.

Referring back to the infinite sequence of terms, we will assume:

$1 \neq 0.999\ldots$

$1 = 0.9 + 0.1$
$1 = 0.99 + 0.01$
$1 = 0.999 + 0.001$
$1 = 0.9999 + 0.0001$
etc.

$1 = 0.999\ldots + \ldots 1$, *(where* …1 *is some infinitesimal quantity, and* 0.999… *is some quantity infinitesimally smaller than* 1*)*

Let κ be substituted for this infinitesimal quantity …1 such that:

$\kappa = \ldots 1$

$1 = 0.999\ldots + \ldots 1$

$1 = (1 - \kappa) + (\kappa)$

Let κ have the property of being a 'smallest possible quantity greater than zero', such that:

$$1 / \infty = \kappa, \qquad 1 / \kappa = \infty, \qquad \kappa \times \infty = 1, \qquad \infty / \infty = 1, \qquad \kappa / \kappa = 1$$

Replacing $\aleph$ for ∞, let $\aleph$ have the property of being a 'greatest possible quantity', so that:

$$1 / \aleph = \kappa, \qquad 1 / \kappa = \aleph, \qquad \kappa \times \aleph = 1, \qquad \aleph / \aleph = 1, \qquad \kappa / \kappa = 1$$

To avoid the regular arguments which dismiss the consistency of the concept of infinitesimals in mathematics, we will assume that κ is a set of infinitesimals such that any conceived quantity smaller than κ and greater than 0 is a member and/or subset of κ.

$$\kappa \{ \kappa_0 , \kappa_1 , \kappa_2 , \kappa_3 , \kappa_4 , \dots , \kappa_n \} \qquad \text{where } \kappa_n \geq 0 \quad \text{and} \quad \kappa_{n-1} > \kappa_n$$

κ_0 is some infinitesimal quantity whose value is the least lower bound of the set of infinitesimals κ, and, at the limit, κ_n is some infinitesimal quantity whose value is the most lower bound of the set of infinitesimals κ.

$$\kappa \{ \kappa_0 \rightarrow \kappa_n \}$$

We will assume that $\aleph$ is a set of infinites or cardinal numbers such that any conceived 'greatest possible quantity' that is infinite and less than C, where C is the continuum, is a member and/or subset of $\aleph$.

$$\aleph \{ \aleph_0 , \aleph_1 , \aleph_2 , \aleph_3 , \aleph_4 , \dots , \aleph_n \} \qquad \text{where } \aleph_n < C\,^* \quad \text{and} \quad \aleph_{n-1} < \aleph_n$$

* C *cannot be a member of the set* $\aleph$ *because* C *is the greatest of all sets and* $\aleph$ *is a subset of* C.

$\aleph_0$ is the set of all natural numbers and $\aleph_0$ is some infinite quantity whose value is the least upper bound of the set of infinites $\aleph$, and, at the limit, $\aleph_n$ is some infinite quantity whose value is the most upper bound of the set of infinites $\aleph$.

$$\aleph \{ \aleph_0 \rightarrow \aleph_n \} \qquad \text{where } \aleph_0 \{ 1, 2, 3, 4, \dots , n \}$$

1.4- Roots of Infinites and Infinitesimals

Let $\forall$ and Ω be extra-finite quantities that intermediate between the finites and the infinites (Ω), and between the finites and the infinitesimals ($\forall$).

$\forall$ is the set of roots for κ_0.

$$\forall \{ \forall_0 , \forall_1 , \forall_2 , \forall_3 , \forall_4 , \dots , \forall_n \} \quad \text{where } \forall_n \leq 1 \quad \text{and} \quad \forall_{n-1} < \forall_n$$

Let the square root of κ_0 be some extra-finite quantity $\forall_1$, where:

$$\forall_0 = \kappa_0^{1} , \qquad \forall_1 = \kappa_0^{1/2} , \qquad \forall_2 = \kappa_0^{1/3}, \qquad \text{etc.}$$

$$\forall_n = \kappa_0^{1/\aleph_0} = \sqrt[\aleph_0]{\kappa_0} = \kappa_0^{\kappa_0} = 1 - \kappa_0$$

Ω is the set of roots for $\aleph_0$.

$$\Omega\ \{\ \Omega_0 , \Omega_1 , \Omega_2 , \Omega_3 , \Omega_4 , \ldots , \Omega_n\ \}\ \text{where } \Omega_n \geq 1 \quad \text{and} \quad \Omega_{n-1} > \Omega_n$$

Let the square root of $\aleph_0$ be some extra-finite quantity Ω_1, where:

$$\Omega_0 = \aleph_0^{\,1}, \qquad \Omega_1 = \aleph_0^{\,1/2}, \qquad \Omega_2 = \aleph_0^{\,1/3}, \qquad \text{etc.}$$

$$\Omega_n = \aleph_0^{\,1/\aleph_0} = \sqrt[\aleph_0]{\aleph_0} = \aleph_0^{\,\kappa_0} = 1 + \kappa_0$$

Assuming that $\forall_0$ through $\forall_n$ are reciprocals of Ω_0 through Ω_n for all $\forall$ and Ω, we find:

$$1 / \Omega_0 = \forall_0, \quad 1 / \forall_0 = \Omega_0, \quad \forall_0 \times \Omega_0 = 1, \quad \Omega_0 / \Omega_0 = 1, \quad \forall_0 / \forall_0 = 1$$

$$1 / \Omega_n = \forall_n, \quad 1 / \forall_n = \Omega_n, \quad \forall_n \times \Omega_n = 1, \quad \Omega_n / \Omega_n = 1, \quad \forall_n / \forall_n = 1$$

(see Appendix)

Though $\forall$ and Ω are extra-finite, since they exist nearer to the infinites and infinitesimals more than ordinary finite numbers and give that $\forall_0 = \kappa_0$ and $\Omega_0 = \aleph_0$, we will assume:

$$\aleph_0 \times \forall_0 = 1; \quad \text{since } \aleph_0 \times \kappa_0 = 1 \qquad\qquad \aleph_0 \times \Omega_0 \approx \aleph_0; \ \text{since } \aleph_0 \times \aleph_0 \approx \aleph_0$$

$$\aleph_0 \times \forall_n = \aleph_0; \ \text{since } \aleph_0 \times 1 = \aleph_0 \qquad\qquad \aleph_0 \times \Omega_n = \aleph_0; \ \text{since } \aleph_0 \times 1 = \aleph_0$$

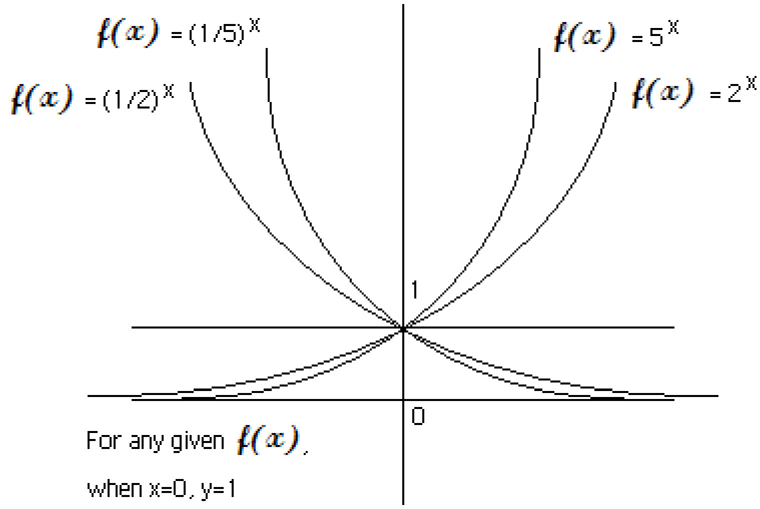
We will refer to the infinites, infinitesimals, and extra-finites as transfinite quantities collectively speaking.

1.5- Arithmetical Treatments of Infinites and Infinitesimals

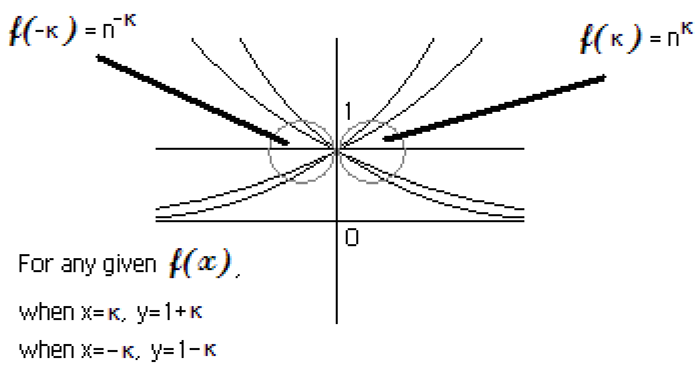
With a calculator, or a pen and much scratch paper, one can obtain the following results:

$$\surd 1 = 1$$
$$\surd 2 \approx 1.41\ldots$$
$$\surd 3 \approx 1.73\ldots$$
$$\surd\surd 4 \approx 1.41\ldots$$
$$\surd\surd\surd 6 \approx 1.251\ldots$$
$$\surd\surd\surd\surd\surd\surd\surd 14 \approx 1.0208\ldots$$
$$\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd 30 \approx 1.000104\ldots$$
$$\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd\surd 134 \approx 1.00000000456\ldots$$

If you carry out an excess of square roots upon any quantity, the results are all the same: the quantity goes towards one plus some very small decimal number.



If one examines a graphic representation of exponential curves, it can be shown that any quantity greater than zero, raised to the power of zero, equals one. Looking at the same graph, it can be argued that any quantity greater than zero, raised to the power of a 'least possible value greater than zero', equals one and some addition (or subtraction) of a very small quantity.



It can be hypothesized that any finite quantity greater than one raised to the infinitesimal root equals one plus some infinitesimal:

$\sqrt[\infty]{n} = n^{\kappa} = 1 + \kappa$ where $n > 1 + \kappa$

(see Appendix)

Assume:

$n^{\kappa} > n^{0} > n^{-\kappa}$, *where* n *is any finite number greater than* 1,

$n^{\kappa} < n^{0} < n^{-\kappa}$, *where* n *is any finite number greater than* 0 *and less than* 1.

Let:

$^{\infty}\sqrt{n} = n^{\kappa} = 1+\kappa$, $n^0 = 1$, $^{-\infty}\sqrt{n} = n^{-\kappa} = 1-\kappa$, when $n^{\kappa} > n^0 > n^{-\kappa}$

and

$1-\kappa = 0.999\ldots$, where 0.999… is the least number greater than 1.

It follow that either:

(a) $^{\infty}\sqrt{\infty} = 1 + \kappa$

(b) $^{\infty}\sqrt{\infty} = \infty + \kappa = \infty$

Before we solve for (a) or (b) we must first continue to define κ_0 and $\aleph_0$ algebraically.

Let:

$\kappa_0 + \kappa_0 = 2\kappa_0$
$\kappa_0 + \kappa_0 + \kappa_0 = 3\kappa_0$
$\kappa_0 + \kappa_0 + \kappa_0 + \kappa_0 = 4\kappa_0$
etc.
$\kappa_0 + \kappa_0 + \kappa_0 +\ldots+ \kappa_0 = \infty \times \kappa_0 = 1$

Let:

$\kappa_0 \times \kappa_0 = \kappa_0^2$
$\kappa_0 \times \kappa_0 \times \kappa_0 = \kappa_0^3$
$\kappa_0 \times \kappa_0 \times \kappa_0 \times \kappa_0 = \kappa_0^4$
etc.
$\kappa_0 \times \kappa_0 \times \kappa_0 \times\ldots\times \kappa_0 = \kappa_0^{\infty} = \kappa_1$

Let:

$\aleph_0 + \aleph_0 = 2\aleph_0$
$\aleph_0 + \aleph_0 + \aleph_0 = 3\aleph_0$
$\aleph_0 + \aleph_0 + \aleph_0 + \aleph_0 = 4\aleph_0$
etc.
$\aleph_0 + \aleph_0 + \aleph_0 +\ldots+ \aleph_0 = \aleph_0 \times \aleph_0 = \aleph_0^2$

Let:

$\aleph_0 \times \aleph_0 = \aleph_0^2$
$\aleph_0 \times \aleph_0 \times \aleph_0 = \aleph_0^3$
$\aleph_0 \times \aleph_0 \times \aleph_0 \times \aleph_0 = \aleph_0^4$
etc.
$\aleph_0 \times \aleph_0 \times \aleph_0 \times\ldots\times \aleph_0 = \aleph_0^{\infty} = \aleph_1$

(From here on, let ∞ be synonymous with $\aleph_0$.)

Dismissing (b) as the solution and choosing to go with (a), $^{\infty}\sqrt{\infty} = 1 + \kappa$, we find:

$$^{\infty}\sqrt{\infty} = {}^{\aleph_0}\sqrt{\aleph_0} = \aleph_0^{\kappa_0} = 1 + \kappa_0$$

1.6- Range of Value

If we treat the equation '$\sqrt[\infty]{\infty} = 1 + \kappa$' algebraically, we find:

$$\sqrt[\infty]{\infty} = 1 + \kappa$$

$$\infty = (1 + \kappa)^{\infty}$$

Substituting $\aleph_0$ for ∞, if we attempt to factor out the equation $(1 + \kappa_0)^{\aleph_0}$ we find a minimum, mean, and maximum range of solutions. These varying solutions are derived by applying algebraic rules to the foregoing definitions of the infinites and infinitesimals.

$$(1 + \kappa_0)^0 = 1$$

$$(1 + \kappa_0)^1 = 1 + \kappa_0$$

$$(1 + \kappa_0)^2 = 1 + 2\,\kappa_0 + \kappa_0^2$$

$$(1 + \kappa_0)^3 = 1 + 3\,\kappa_0 + 3\,\kappa_0^2 + \kappa_0^3$$

$$(1 + \kappa_0)^4 = 1 + 4\,\kappa_0 + 6\,\kappa_0^2 + 4\,\kappa_0^3 + \kappa_0^4$$

etc.

Examining the terms in our series as it approaches $\aleph_0$, we discover that there are multiple ways of factoring an answer. Examining the final terms of the sequence first running the summation sequence in a reverse direction, opposite from usual, we can get:

$$(1 + \kappa_0)^{\aleph_0} = 1 + \aleph_0\,\kappa_0^{\aleph_0} + \aleph_0\,\kappa_0^{\aleph_0 - 1} + \aleph_0\,\kappa_0^{\aleph_0 - 2} + \ldots$$

$$(1 + \kappa_0)^{\aleph_0} = 1 + \aleph_0\,\kappa_1 + \aleph_0\,\kappa_1 + \aleph_0\,\kappa_1 + \ldots$$

$$(1 + \kappa_0)^{\aleph_0} = 1 + \kappa_0 + \kappa_0 + \kappa_0 + \ldots$$

$$(1 + \kappa_0)^{\aleph_0} = 1 + \{\kappa_0 + \kappa_0 + \kappa_0 + \ldots\}$$

In the same way that subtracting finite quantities from infinite quantities does not change the overall quantity of the infinite, adding two infinitesimal does not seem to really increase the value; you still have an infinitesimal quantity, such that:

$$2\kappa_0 \approx \kappa_0\,.$$

Similarly, raising an infinitesimal to the power of a finite does not really change its value:

$$\kappa_0^2 \approx \kappa_0.$$

Therefore the minimum solution is:

$$= 1 + \{\kappa_0 + \kappa_0 + \kappa_0 + \ldots\}$$

$$= 1 + \text{some infinitesimal quantity} > \kappa_0$$

If we recall that we have presumed the definition: $\kappa_0 \times \aleph_0 = 1$, we can get a mean solution:

$$= 1 + \{ \kappa_0 + \kappa_0 + \kappa_0 + \ldots \}$$

$$= 1 + \{ \aleph_0 \times \kappa_0 \} = 1 + 1$$

$$= 2$$

However, if we run the summation in the regular direction we find:

$$= 1 + \aleph_0\, \kappa_0 + \aleph_0\, \kappa_0^2 + \aleph_0\, \kappa_0^3 + \aleph_0\, \kappa_0^4 + \ldots$$

$$= 1 + \underbrace{\aleph_0\, \kappa_0}_{1} + \underbrace{\aleph_0\, \kappa_0^2}_{1} + \underbrace{\aleph_0\, \kappa_0^3}_{1} + \underbrace{\aleph_0\, \kappa_0^4}_{1} + \ldots$$

$$= 1 + 1 + 1 + 1 + 1 + \ldots$$

$$= \aleph_0$$

Thus, for this equation, there is a minimum, mean, and maximum solution depending on how you factor and run the summation of the terms generated.

$$(1 + \kappa_0)^{\aleph_0}$$

Minimum: $= 1$ + some infinitesimal quantity $> \kappa_0$

Mean: $= 2$

Maximum: $= \aleph_0$

If it can be determined that a quantity is either infinite or infinitesimal, it can be shown that when treated algebraically it exhibits a property of having a range of value rather than a specific value. Therefore, if a quantity is infinite or infinitesimal in nature, and that quantity exists over a range between a minimum (compressed) form and a maximum (expanded) form, then the quantity *perambulates* on the continuum.

Compression of addition of initial infinitesimals yields an expanded initial infinitesimal:

$$\kappa_0 + \kappa_0 + \kappa_0 + \kappa_0 + \ldots = \kappa_{0_e}$$ *where* κ_{0_e} *is the expanded form of* κ_0.

Expansion of addition of initial infinitesimals yields one:

$$\kappa_0 + \kappa_0 + \kappa_0 + \kappa_0 + \ldots = 1$$

Compressed form of the initial infinite set; the set of all natural numbers:

$$1 + 1 + 1 + 1 + \ldots = \aleph_{0_c}$$ *where* $\aleph_0$ is $\{ 1, 2, 3, 4, \ldots \}$

1.7- Perambulation

Infinite and/or infinitesimal quantities exhibit the property of perambulating along a range of possible values; between a minimum compressed form and a maximum expanded form depending upon how the infinite and/or infinitesimal quantities are treated algebraically.

Therefore, the following states:

$$(1 + \kappa_0)^{\aleph_0} \ \{ (1 + \kappa_{0_e}) \longrightarrow \aleph_{0_c} \}$$

"*The equation* $(1 + \kappa_0)^{\aleph_0}$ *perambulates over the range of* $(1 + \kappa_{0_e})$ *and* $\aleph_{0_c}$ *where* $(1 + \kappa_{0_e})$ *is the minimum term, and* $\aleph_{0_c}$ *is the maximum term.*"

In other words, a finite number greater than one, less than two, when raised to the power of $\aleph_0$ in compressed and expanded forms, perambulates between two plus the expanded form of the initial infinitesimal and the compressed form of the set of all natural numbers.

$$(1 + \kappa_0)^{\aleph_{0_c}} = 1 + \kappa_{0_e}$$

$$(1 + \kappa_0)^{\aleph_{0_e}} = \aleph_{0_c}$$

$$(1 + \kappa_0)^{\aleph_0} \ \{ (1 + \kappa_0)^{\aleph_{0_c}} \rightarrow (1 + \kappa_0)^{\aleph_{0_e}} \}$$

Therefore, $\aleph_0$, the set of all natural numbers, is the initial infinite or cardinal number whose quantity ranges (perambulates) between a compressed and expanded form.

$$\aleph_0 \ \{ \aleph_{0_c} \longrightarrow \aleph_{0_e} \}$$

We will let the power set of the compressed form of the initial infinite number be the expanded form of the same, a finite number greater than or equal to two, when raised to the power of the compressed form of $\aleph_0$, equals the expanded form of $\aleph_0$, and since each consecutive infinite is dwarfed by the next in a similar way that finites are dwarfed by $\aleph_0$, we will assume:

$$2^{\aleph_{0_c}} = \aleph_{0_e} \qquad n^{\aleph_{0_c}} = \aleph_{0_e} \qquad \aleph_{0_c}^{\aleph_{0_c}} = \aleph_{0_e}$$

Thus, the expanded expression of $\aleph_0$ perambulates between the power set of $\aleph_0$ compressed and $\aleph_0$ compressed raised to the power of $\aleph_0$ compressed.

$$\aleph_{0_e} \ \{ 2^{\aleph_{0_c}} \longrightarrow \aleph_{0_c}^{\aleph_{0_c}} \}$$

Therefore, we will redefine the powerset of the set of all natural number from:

$$2^{\aleph_0} = \aleph_1 \qquad \text{into:} \qquad 2^{\aleph_{0_e}} = \aleph_{1_c}$$

The powerset of the expanded form of the set of all natural numbers $\aleph_0$ is the compressed form of the next consecutive infinite or cardinal number $\aleph_1$.

1.8- Reciprocating Perambulations

Maintaining a consistency with the algebraic relationships between expanded and compressed forms of infinite and infinitesimal quantities, we find the following expressions to be consistent:

$$1 / \kappa_{0_e} = \aleph_{0_c} \qquad 1 / \aleph_{0_c} = \kappa_{0_e} \qquad \aleph_{0_c} \times \kappa_{0_e} = 1 \qquad \aleph_0 \ \{ \aleph_{0_c} \rightarrow \aleph_{0_e} \}$$

$$1 / \kappa_{0_c} = \aleph_{0_e} \qquad 1 / \aleph_{0_e} = \kappa_{0_c} \qquad \aleph_{0_e} \times \kappa_{0_c} = 1 \qquad \kappa_0 \ \{ \kappa_{0_c} \rightarrow \kappa_{0_e} \}$$

$$1 / \kappa_{1_e} = \aleph_{1_c} \qquad 1 / \aleph_{1_c} = \kappa_{1_e} \qquad \aleph_{1_c} \times \kappa_{1_e} = 1 \qquad \aleph_1 \ \{ \aleph_{1_c} \rightarrow \aleph_{1_e} \}$$

$$1 / \kappa_{1_c} = \aleph_{1_e} \qquad 1 / \aleph_{1_e} = \kappa_{1_c} \qquad \aleph_{1_e} \times \kappa_{1_c} = 1 \qquad \kappa_1 \ \{ \kappa_{1_c} \rightarrow \kappa_{1_e} \}$$

$$1 / \kappa_{2_e} = \aleph_{2_c} \qquad 1 / \aleph_{2_c} = \kappa_{2_e} \qquad \aleph_{2_c} \times \kappa_{2_e} = 1 \qquad \aleph_2 \ \{ \aleph_{2_c} \rightarrow \aleph_{2_e} \}$$

$$1 / \kappa_{2_c} = \aleph_{2_e} \qquad 1 / \aleph_{2_e} = \kappa_{2_c} \qquad \aleph_{2_e} \times \kappa_{2_c} = 1 \qquad \kappa_2 \ \{ \kappa_{2_c} \rightarrow \kappa_{2_e} \}$$

etc.

$$1 / \kappa_{n_e} = \aleph_{n_c} \qquad 1 / \aleph_{n_c} = \kappa_{n_e} \qquad \aleph_{n_c} \times \kappa_{n_e} = 1 \qquad \aleph_n \ \{ \aleph_{n_c} \rightarrow \aleph_{n_e} \}$$

$$1 / \kappa_{n_c} = \aleph_{n_e} \qquad 1 / \aleph_{n_e} = \kappa_{n_c} \qquad \aleph_{n_e} \times \kappa_{n_c} = 1 \qquad \kappa_n \ \{ \kappa_{n_c} \rightarrow \kappa_{n_e} \}$$

1.9- Arithmetical Treatments of Perambulations

Treating these definitions algebraically, we get:

$$(1 + \kappa_0)^{\aleph_{0_c}} = 1 + \kappa_{0_e} \quad \text{and} \quad (1 + \kappa_0)^{\aleph_{0_e}} = \aleph_{0_c}$$

$$1 + \kappa_0 = (1 + \kappa_{0_e})^{\kappa_{0_e}} \qquad\qquad 1 + \kappa_0 = \aleph_{0_c}^{\kappa_{0_c}}$$

Given that the mean solution is:

$$2 = (1 + \kappa_0)^{\aleph_0} \quad \text{and} \quad 2^{\kappa_0} = 1 + \kappa_0$$

We can also derive the solutions that:

$$2 = (1 + \kappa_{0_e})^{\aleph_{0_c}} \quad \text{and} \quad 2 = (1 + \kappa_{0_c})^{\aleph_{0_e}}$$

$$2^{\kappa_{0_e}} = 1 + \kappa_{0_e} \qquad\qquad 2^{\kappa_{0_c}} = 1 + \kappa_{0_c}$$

Therefore we will assume:

$$\left.\begin{array}{r} (1+\kappa_0)^{\aleph_{0_c}} = 1+\kappa_{0_e} \\ \text{and} \\ 2^{\kappa_{0_e}} = 1+\kappa_{0_e} \\ 2 = (1+\kappa_{0_e})^{\aleph_{0_c}} \end{array}\right\} 1+\kappa_0 \le (1+\kappa_{0_e})^{\aleph_{0_c}} \le 2$$

$$\left.\begin{array}{r} (1+\kappa_0)^{\aleph_{0_e}} = \aleph_{0_c} \\ \text{and} \\ 2^{\kappa_{0_c}} = 1+\kappa_{0_c} \\ 2 = (1+\kappa_{0_c})^{\aleph_{0_e}} \end{array}\right\} 2 \le (1+\kappa_{0_c})^{\aleph_{0_e}} \le \aleph_{0_c}$$

1.10- Subambulation

Since, in both examples, it is not specified whether or not the κ_0 in $(1 + \kappa_0)$ should be an expression of a compressed or expanded form, and since $\kappa_0 \ \{\kappa_{0_c} \rightarrow \kappa_{0_e}\}$ represents the range between compressed and expanded forms of κ_0, we will assume:

$$(1+\kappa_0)^{\aleph_{0_c}} = 1+\kappa_{0_e} \qquad\qquad (1+\kappa_0)^{\aleph_{0_e}} = \aleph_{0_c}$$

$$1+\kappa_0 = (1+\kappa_{0_e})^{\kappa_{0_e}} \qquad\qquad = \aleph_{0_c}^{\kappa_{0_c}}$$

$$1+\kappa_{0_e}^- = (1+\kappa_{0_e})^{\kappa_{0_e}} \qquad\qquad 1+\kappa_{0_c} = \aleph_{0_c}^{\kappa_{0_c}}$$

$$1+\kappa_{0_e}^- = 1+\kappa_0$$

Letting: $\kappa_{0_c}^+ = \kappa_0 = \kappa_{0_e}^-$ and $\kappa_{0_c}^- < \kappa_{0_c} < \kappa_0 < \kappa_{0_e} < \kappa_{0_e}^+$

Using a concept of sublet ambulations, denoted by a "+" or "-" sign, we can derive that:

$$(1+\kappa_{0_e}^-)^{\aleph_{0_c}} = 1+\kappa_{0_e} \qquad\qquad (1+\kappa_{0_e})^{\aleph_{0_c}} = 1+\kappa_{0_e}^+$$

$$1+\kappa_{0_e}^- = (1+\kappa_{0_e})^{\kappa_{0_e}} \qquad\qquad 1+\kappa_{0_e} = (1+\kappa_{0_e}^+)^{\kappa_{0_e}}$$

$$(1+\kappa_{0_e}^{-})^{\aleph_{0_c}} = (1+\kappa_{0_e}^{+})^{\kappa_{0_e}}$$

$$1+\kappa_{0_e}^{-} = [(1+\kappa_{0_e}^{+})^{\kappa_{0_e}}]^{\kappa_{0_e}}$$

$$1+\kappa_{0_e}^{-} = [\,1+\kappa_{0_e}\,]^{\kappa_{0_e}}$$

(see Appendix)

This leads us into allowing for an endless progression and regression of subambulations of each compressed and expanded form.

$$\begin{array}{cccc}
 & \kappa_0 & & \\
\kappa_{0_c} & & \kappa_{0_e} & \\
\kappa_{0_c}^{-} & \kappa_{0_c}^{+} & \kappa_{0_e}^{-} & \kappa_{0_e}^{+} \\
\kappa_{0_c}^{=} & \kappa_{0_c}^{++} & \kappa_{0_e}^{=} & \kappa_{0_e}^{++} \\
\kappa_{0_c}^{\equiv} & \kappa_{0_c}^{+++} & \kappa_{0_e}^{\equiv} & \kappa_{0_e}^{+++} \\
\vdots & \vdots & \vdots & \vdots \\
\kappa_{0_c}^{\min(-)} & \kappa_{0_c}^{\max(+)} & \kappa_{0_e}^{\min(-)} & \kappa_{0_e}^{\max(+)}
\end{array}
\qquad
\begin{array}{cccc}
 & \aleph_0 & & \\
\aleph_{0_c} & & \aleph_{0_e} & \\
\aleph_{0_c}^{-} & \aleph_{0_c}^{+} & \aleph_{0_e}^{-} & \aleph_{0_e}^{+} \\
\aleph_{0_c}^{=} & \aleph_{0_c}^{++} & \aleph_{0_e}^{=} & \aleph_{0_e}^{++} \\
\aleph_{0_c}^{\equiv} & \aleph_{0_c}^{+++} & \aleph_{0_e}^{\equiv} & \aleph_{0_e}^{+++} \\
\vdots & \vdots & \vdots & \vdots \\
\aleph_{0_c}^{\min(-)} & \aleph_{0_c}^{\max(+)} & \aleph_{0_e}^{\min(-)} & \aleph_{0_e}^{\max(+)}
\end{array}$$

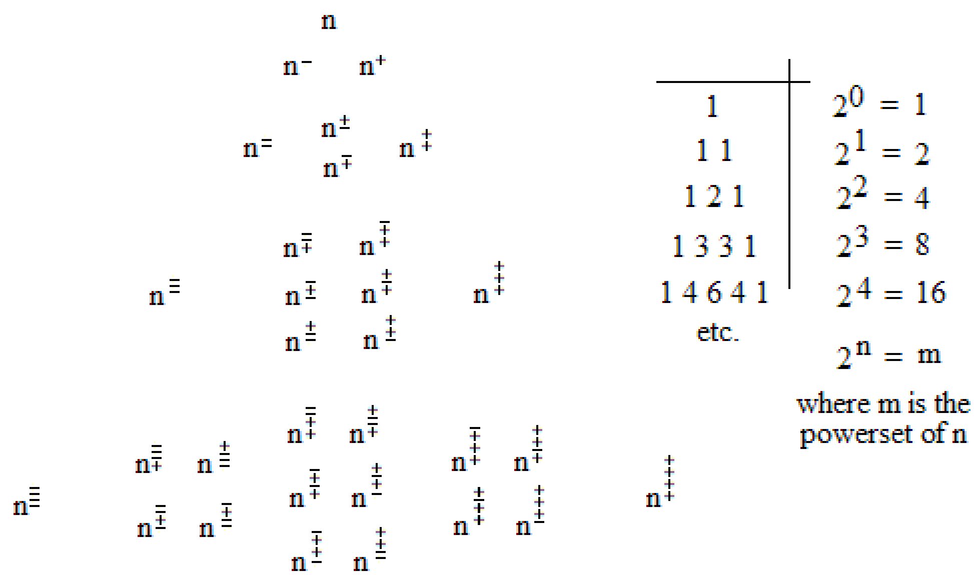
Let "+" or "-" denote an endless progression (+) or an endless regression (-) of sublet perambulations (subambulations) where the maximum and minimum terms are expressed as the progressively most upper-bound (+) or regressively least lower-bound (-) expression of all possible subambulations.

(see Appendix)

Subambulations are derivable for all infinites, infinitesimals, and extra-finite quantities such that they allow for an ordered accounting of the reciprocities between the infinites and infinitesimals and the reciprocal relationships within the extra-finite quantities.*

> *****_It must be noted that because of the infinite potential of diversity in the ambulations of transfinite quantities, there exist algebraic expressions with solutions that exist between the expressions we specifically need for our purposes and that are not possible to solve for without difficultly in deriving needless correct solutions. These are not indeterminate rather inconvenient._

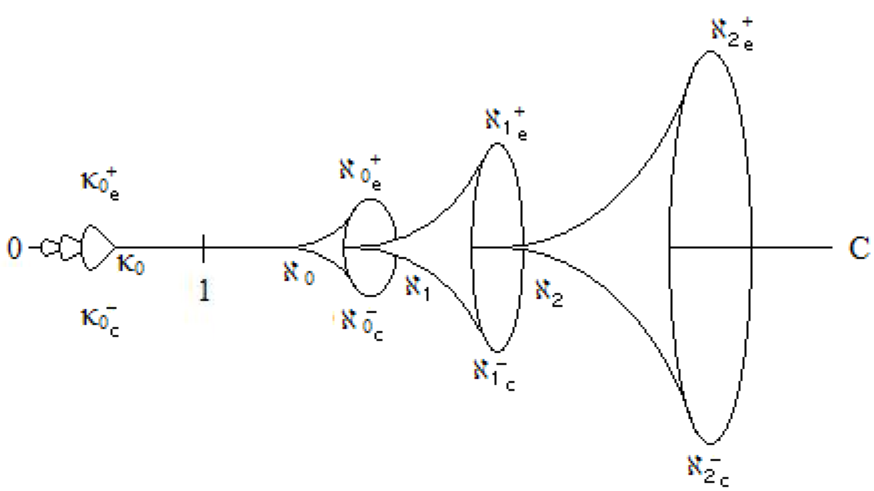
Example: $\quad (\aleph_{0_e}^{-})^{\kappa_{0_e}^{++}} = \ ?$

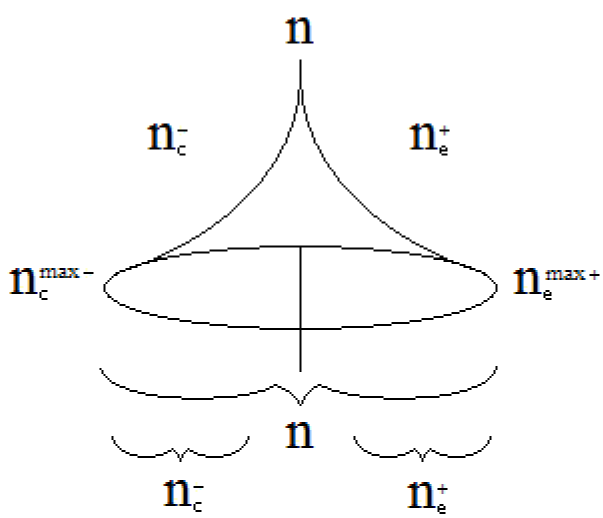


In carrying out the permutations of subambulation, if we let subambulations that share the same degrees of "+" and "-", regardless of order of appearance (since we are dealing with sets), we can say that they overlap and are the same subambulation. In doing so the order of subambulation follows a Pascal's triangle pattern where each level of subambulation configures to corresponding levels of a Pascal's triangle.

1.11- Ambulation

It has be shown that together both perambulation and subambulation maintain consistent arithmetical treatments of infinites with infinitesimals having reciprocal relationships. Perambulation and subambulation are dual aspects of *ambulation*, the variable range of communicable values derived when treating infinites with infinitesimals algebraically. The idea of perambulation arose from the binomial function $(a + b)^n$, where $a = 1$, $b = \kappa_0$, and $n = \aleph_0$, and the idea of subambulation follows the number sequence of Pascal's triangle. Thus both reveal an underlying pattern of consistency with conventional arithmetic. Ambulation allow infinites and infinitesimals to be treated algebraically as specific values using functions that equal sets of possible values where each member of each set accounts for different specific ambulations and reciprocations thereof.





$$n\ \{n_c \to n_e\}$$

$$n_c\ \{n_c^- \to n_c^+\}$$

$$n_e\ \{n_e^- \to n_e^+\}$$

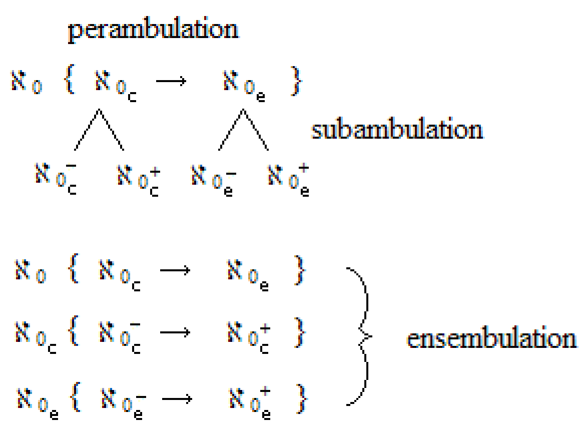
$$n_c^{max+} = n = n_e^{max-}$$

Equivalences can be derived between the greatest subambulation of a compressed perambulation, the least subambulation of the expanded perambulation, and the initial ambulating quantity.

$$n_c^{max+} = n = n_e^{max-}$$

The infinite relationships between the perambulations/subambulations of each infinite and each infinitesimal is another degree to ambulation that shall be call an *ensembulation*. An ordered sequence of ensembulations shall be called a *compulation*.

An ensembulation is the collective expression of some transfinite quantity whose value ambulates over a range of possible expressions between a variety of degrees of minimum/maximum compressed and minimum/maximum expanded forms.



A compulation is an ordered series of transfinite numbers ranging from an initial to a final value where the range of quantities encompass an enembulation of ambulations; such as the range from the least upper bound infinite $\aleph_0$ (the set of all natural numbers) to the most upper bound infinite $\aleph_n$ (the power set least the set of the continuum)*.

**The continuum cannot be a member of the cardinal continuum because the cardinal continuum is a subset of the continuum*.

$$\text{compulation} \left\{ \begin{array}{ll} \aleph_0 & \{\aleph_{0_c} \rightarrow \aleph_{0_e}\} \\ \aleph_1 & \{\aleph_{1_c} \rightarrow \aleph_{1_e}\} \\ \aleph_2 & \{\aleph_{2_c} \rightarrow \aleph_{2_e}\} \\ \vdots & \quad \text{etc.} \\ \aleph_n & \{\aleph_{n_c} \rightarrow \aleph_{n_e}\} \end{array} \right.$$

The gelatinous nature of C3 captures the essence of the continuum incorporating the concept of ambulation through the definitions of perambulation, subambulation, ensembulation, and compulation that later have applications in Non-Standard Analysis and Infinitesimal Calculus. We can define the infinitesimals, finites, extra-finites, infinites, and zero in terms of a continuum ranging from the minimum compressed form of the continuum to the maximum expanded form of the same.

1.12- Central Core Values and Super Order

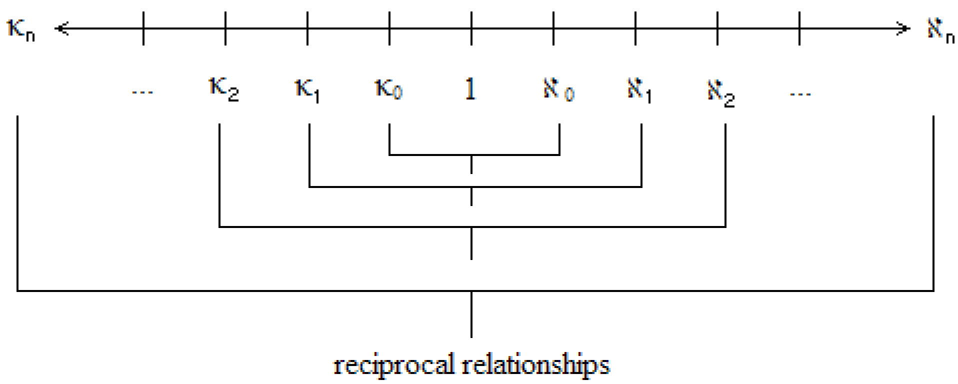
Central core values are terms that off-shoots from a ambulation of a transfinite number that, when treated algebraically, exhibit mathematic logical consistency. A transfinite term can be used algebraically if and only if the transfinite term is a *central core value* and/or the origin of a particular ambulation such as the transfinites we have used in the arithmetical treatments.

Not all ambulations of a transfinite quantity, when treated algebraically, easily exhibit mathematic logical consistency. Though there exist derivable inexpressive algebraic expressions as gaps in completeness, there is an order to this incompleteness that actually serves for a particular kind of count-ability. Ambulation deflates the uncountable aloof nature of transfinite quantities into being comprehensive ordered sequences. Though we cannot account for each and every algebraic expression, we can account for the inexpressive algebraic expressions by way of sets of *super order*.

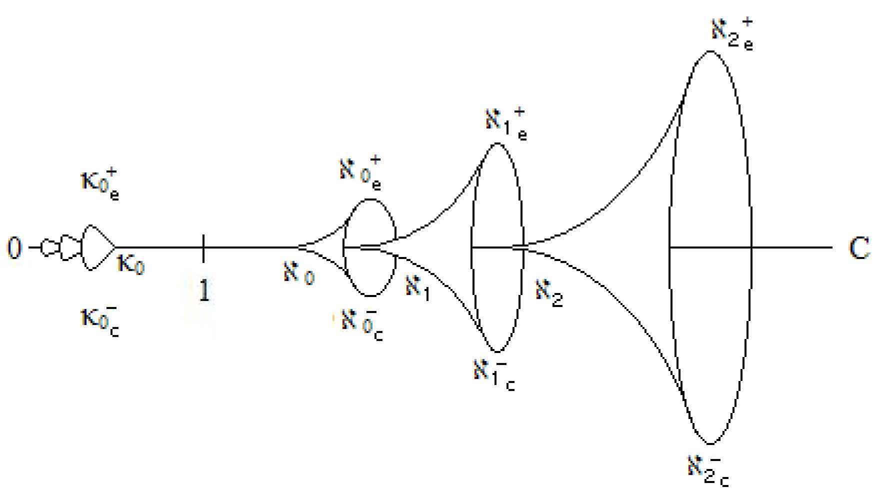
Super order is an unintelligible interval of order which cannot be placed into a 1-1 correspondence with any countable set, such as the set of all natural numbers, and can only be place in a correspondence with the infinite term whose power set is the continuum (*see section* 2.0).

Referring to a cardinal continuum, we find an infinitude of infinitesimals, extra-finite, and infinite numbers in “both” directions in 1-1 reciprocal relationships with each other gapped by intervals of super order.

Cardinal Continuum



We also extended the cardinal continuum in another dimension of the reciprocal relationships between the perambulations and subambulations of trans-finite numbers.



1.13- Defining the Reciprocal of Zero

However, this leaves us with an inevitable problem:

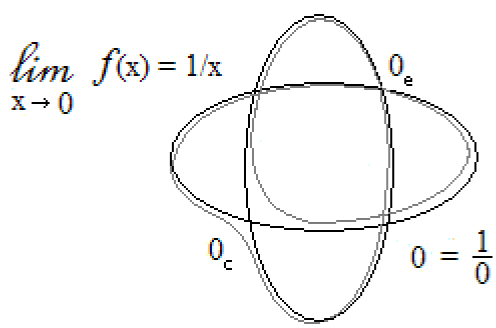
> *If we let the final term of set κ be equal to zero, and the final term of set κ is the reciprocal of the final term of set ℵ, then what is the final term of set ℵ?*

In other words:

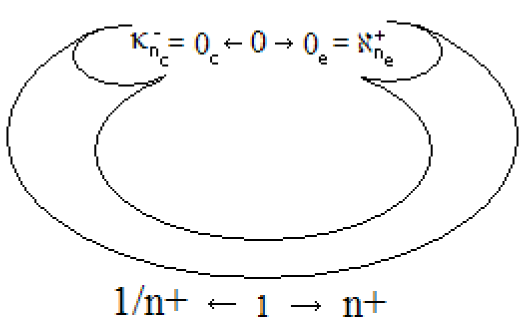
> ***What is the solution to 1/0?*** ***What is the reciprocal of zero?***

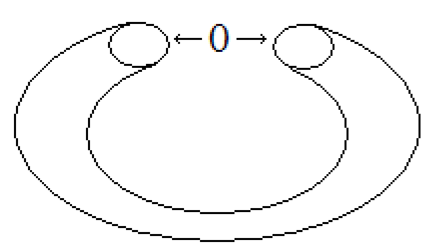
The answer lies in the fact that as do the infinitesimals, the infinites regress and progress indefinitely. It should follow that, given the 'indefiniteness' concerning both infinite and infinitesimal numbers, what ever is deduced at one end of the continuum should correspond to what ever is deduced as the other end. So it logically follows that what ever is at the end of one side of the continuum should have a reciprocal at the other end.

We already know that given the infinitude of both finite and transfinite numbers, all numbers can be accounted for as having a reciprocal. This leave no other possible number as being the reciprocal of zero other than zero itself. In some way, zero is at both ends of the continuum, i.e. the continuum in all directions ends at zero.



Like the infinites and infinitesimals, we can induce the idea that zero ambulates with the exception that zero ambulates the entire continuum to the extent that zero is the continuum. This is not to say that there are two zeros, but rather that in two separate ways, zero exists at both extreme ends of the continuum as one end. We will differentiate zero on the infinitesimal end of the continuum from zero on the infinite end of the continuum where 0_c represents the compressed or common form of zero, and 0_e represents the expanded or infinite form of zero.





1.14- Un-ordinals and the Non-set

Let us assume for now:

$0 \times 0 = 0 \qquad 1 \times 0 = 0 \qquad 0/1 = 0 \qquad 0/n = 0$

$\mathbf{0/0 = 1} \qquad \mathbf{1/0 = 0} \qquad \mathbf{n/0 = 0}$

If we begin to play with the definitions algebraically, we quickly find some apparent problems such as false equalities:

$$
\begin{aligned}
0/0 &= 1 \\
0/0 &= 0 \times 1/0 = 1 \\
&= 0 \times 0 = 1 \\
&= 0 = 1 \\
&\quad 0 = 1
\end{aligned}
$$

Or even more simply by using substitution:

$1/0 = 0$

$1 = 0 \times 0$

Where n is any given number, we end up with the additional apparent problem of:

$n/0 = 0$

$n = 0 \times 0$

When zero is treated algebraically, it can be used to derive any number or false equality. What at first appears to be a contradiction becomes the key to the final part of the solution to the continuum hypothesis. When put this into an ordered sequence, we get:

$0/0 = 0 \times 0 = 0$

$0/0 = 0 \times 0 = 1$

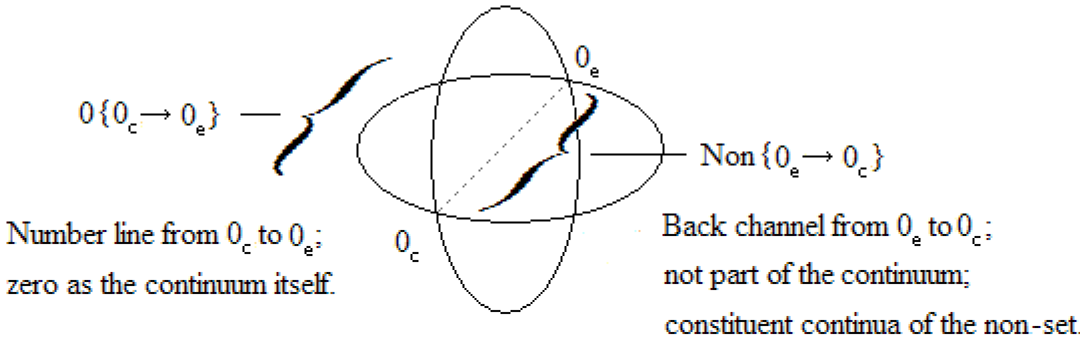
$0/0 = 0 \times 0 = 2$

etc.

$0/0 = 0 \times 0 = n$

Thus, the product of zero multiplied by itself and/or divided by itself derives any possible number. Here, we come to a new concept of the set of *un-ordinals*: a non-set of nonsensical false equalities and indeterminate forms that, as a background set of rejects excluded from the continuum, serve as a collecting bin of mathematical impossibilities.

For every ordinal there is a continua of un-ordinals and there are no countable sets of un-ordinals. Let the set of un-ordinals be expressed existing as the non-set lying outside the set of all numbers as though through a “back (hidden) channel” between the expanded expression 0_e and the compressed expression 0_c:



$$C\ \{\{0\},\ \{0_c \rightarrow 0_e\}\}$$

The un-ordinals can be expressed through ordered sequences of false equalities. Recall:

$0/0 = 0x0 = 0$
$0/0 = 0x0 = 1$
$0/0 = 0x0 = 2$
$0/0 = 0x0 = 3$
etc.

Let this series be a subset of the un-ordinals. The number of un-ordinals is uncountable and they exist outside the sets of the real, hyper-real, ordinal, et al. There are also false equalities which can be placed into a 1-1 with these un-ordinals.

1+1 = 0	—	0/0 = 0x0 = 0	1+0 = 0	—	0/0 = 0x0 = 0
1+1 = 1	—	0/0 = 0x0 = 1	1+1 = 0	—	0/0 = 0x0 = 1
1+1 = 2	—	0/0 = 0x0 = 2	1+2 = 0	—	0/0 = 0x0 = 2
1+1 = 3	—	0/0 = 0x0 = 3	1+3 = 0	—	0/0 = 0x0 = 3
	etc.			etc.	
1+1 = n	—	0/0 = 0x0 = n	1+n = 0	—	0/0 = 0x0 = n

Un-ordinals expressed as nonsensicals or false equalities are solutions to equations that are correct solution to incorrect equations and/or incorrect solutions to correct equation, such that no other equation can give such solution, and no other solution exists for such equation, yet it is not commutable between the equation and solution. These make up a significant substructure of mathematical reality that can be sequenced as continua subsets of the non-set, which later prove to be useful in determining the indeterminate forms.

1.15- Counting the Continuum using Power Sets

Starting out with the smallest possible member of set κ having an equivalence with 0_c, we will count the entire continuum from end to end, from zero to zero, we will proceed to count through the entire continuum.

$$2^{0_c} = 1 + 0_c$$

$$\updownarrow \qquad \} \quad 0_c = \kappa_{n_c}^-$$

$$\left.\begin{array}{l} 2^{\kappa_{n_c}^-} = 1 + \kappa_{n_c}^- \\ 2^{\kappa_{n_c}^+} = 1 + \kappa_{n_c}^+ \end{array}\right\} \begin{array}{l} 2^{\kappa_{n_c}} = 1 + \kappa_{n_c} \\ \kappa_{n_c}\ \{\ \kappa_{n_c}^- \longrightarrow \kappa_{n_c}^+\ \} \end{array}$$

$$\}\ \kappa_{n_c}^+ = \kappa_{n_e}^- = \kappa_n \qquad \left.\right\} \begin{array}{l} 2^{\kappa_n} = 1 + \kappa_n \\ \kappa_n\ \{\ \kappa_{n_c} \longrightarrow \kappa_{n_e}\ \} \end{array}$$

$$\left.\begin{array}{l} 2^{\kappa_{n_e}^-} = 1 + \kappa_{n_e}^- \\ 2^{\kappa_{n_e}^+} = 1 + \kappa_{n_e}^+ \end{array}\right\} \begin{array}{l} 2^{\kappa_{n_e}} = 1 + \kappa_{n_e} \\ \kappa_{n_e}\ \{\ \kappa_{n_e}^- \longrightarrow \kappa_{n_e}^+\ \} \end{array}$$

$$\vdots \qquad \}\ \kappa_{n_e}^{\max +} \text{ and } \kappa_{n_c\text{-}1}^{\min -}$$

$$\left.\begin{array}{l} 2^{\kappa_{n_c}^- \text{-}1} = 1 + \kappa_{n_c}^- \text{-}1 \\ 2^{\kappa_{n_c}^+ \text{-}1} = 1 + \kappa_{n_c}^+ \text{-}1 \end{array}\right\} \begin{array}{l} 2^{\kappa_{n_c\text{-}1}} = 1 + \kappa_{n_c\text{-}1} \\ \kappa_{n_c\text{-}1}\ \{\kappa_{n_c}^- \text{-}1 \longrightarrow \kappa_{n_c}^+ \text{-}1\} \end{array}$$

$$\}\ \kappa_{n_c}^+ \text{-}1 = \kappa_{n_e}^- \text{-}1 = \kappa_{n\text{-}1} \qquad \left.\right\} \begin{array}{l} 2^{\kappa_{n_e\text{-}1}} = 1 + \kappa_{n_e\text{-}1} \\ \kappa_{n\text{-}1}\ \{\ \kappa_{n_c\text{-}1} \longrightarrow \kappa_{n_e\text{-}1}\ \} \end{array}$$

$$\left.\begin{array}{l} 2^{\kappa_{n_e}^- \text{-}1} = 1 + \kappa_{n_e}^- \text{-}1 \\ 2^{\kappa_{n_e}^+ \text{-}1} = 1 + \kappa_{n_e}^+ \text{-}1 \end{array}\right\} \begin{array}{l} 2^{\kappa_{n_e\text{-}1}} = 1 + \kappa_{n_e\text{-}1} \\ \kappa_{n_e\text{-}1}\ \{\kappa_{n_e}^- \text{-}1 \longrightarrow \kappa_{n_e}^+ \text{-}1\} \end{array}$$

$$\vdots \qquad \}\ \kappa_{n_e\text{-}1}^{\max +} \text{ and } \kappa_{n_c\text{-}2}^{\min -}$$

$\kappa_{n_c}^+ = \kappa_{n_e}^-$ represents a point of *inner-ambulation* continuity where the least compressed form of a transfinite number equals the least expanded form of the same. Thus, these extreme subambulation are also equivalent to the initial: $\kappa_{n_c}^+ = \kappa_{n_e}^- = \kappa_n$.

$\kappa_{n_e}^{\max +}$ and $\kappa_{n_c\text{-}1}^{\min -}$ represents two points of *extra-ambulation* continuity where the most expanded form of an infinitesimal meets the most compressed form of the next.* A single point can only be as small as the most lower bound infinitesimals greater than zero, but an interval of infinitesimals can be reduced to zero.

*Infinitesimals are decimal numbers and as roots do not yield power sets where as infinites as roots can yield power sets.

$$\left.\begin{aligned} 2^{\kappa_{n_c}} &= 1+\kappa_{n_c} \\ 2^{\kappa_{n_e}} &= 1+\kappa_{n_e} \end{aligned}\right\} \begin{aligned} &2^{\kappa_n} = 1+\kappa_n \\ &\kappa_n\{\kappa_{n_c} \longrightarrow \kappa_{n_e}\} \end{aligned}$$

$$\left.\begin{aligned} 2^{\kappa_{n_c}-1} &= 1+\kappa_{n_c-1} \\ 2^{\kappa_{n_e}-1} &= 1+\kappa_{n_e-1} \end{aligned}\right\} \begin{aligned} &2^{\kappa_{n-1}} = 1+\kappa_{n-1} \\ &\kappa_{n-1}\{\kappa_{n_c-1} \longrightarrow \kappa_{n_e-1}\} \end{aligned}$$

$$\left.\begin{aligned} 2^{\kappa_{n_c}-2} &= 1+\kappa_{n_c-2} \\ 2^{\kappa_{n_e}-2} &= 1+\kappa_{n_e-2} \end{aligned}\right\} \begin{aligned} &2^{\kappa_{n-2}} = 1+\kappa_{n-2} \\ &\kappa_{n-2}\{\kappa_{n_c-2} \longrightarrow \kappa_{n_e-2}\} \end{aligned}$$

$$\downarrow \; \uparrow \qquad \Big\} \text{ super order}$$

$$\left.\begin{aligned} 2^{\kappa_{2_c}} &= 1+\kappa_{2_c} \\ 2^{\kappa_{2_e}} &= 1+\kappa_{2_e} \end{aligned}\right\} \begin{aligned} &2^{\kappa_2} = 1+\kappa_2 \\ &\kappa_2\{\kappa_{2_c} \longrightarrow \kappa_{2_e}\} \end{aligned}$$

$$\left.\begin{aligned} 2^{\kappa_{1_c}} &= 1+\kappa_{1_c} \\ 2^{\kappa_{1_e}} &= 1+\kappa_{1_e} \end{aligned}\right\} \begin{aligned} &2^{\kappa_1} = 1+\kappa_1 \\ &\kappa_1\{\kappa_{1_c} \longrightarrow \kappa_{1_e}\} \end{aligned}$$

$$\left.\begin{aligned} 2^{\kappa_{0_c}} &= 1+\kappa_{0_c} \\ 2^{\kappa_{0_e}} &= 1+\kappa_{0_e} \end{aligned}\right\} \begin{aligned} &2^{\kappa_0} = 1+\kappa_0 \\ &\kappa_0\{\kappa_{0_c} \longrightarrow \kappa_{0_e}\} \end{aligned}$$

$$\Bigg\} \; \kappa\{\kappa_0 \longrightarrow \kappa_n\}$$

Therefore, it can be shown that the set κ, set of all infinitesimals, ambulates over a range from the least lower bound infinitesimal set κ_0 to the most lower bound infinitesimal set κ_n; from the initial infinitesimal to the null (empty) set. Thus, the compressed or common form of zero (null set) is a member of the set κ as its most lower bound member.

$$\kappa\{\kappa_0 \longrightarrow 0_c\}$$

$$\kappa\{\kappa_0 \longrightarrow \kappa_n\} \qquad \kappa_0\{\kappa_{0_c} \longrightarrow \kappa_{0_e}\} \qquad \kappa_{0_c}\{\kappa_{0_c}^- \longrightarrow \kappa_{0_c}^+\}$$

$$\kappa_n\{\kappa_{n_c} \longrightarrow \kappa_{n_e}\} \qquad \kappa_{0_e}\{\kappa_{0_e}^- \longrightarrow \kappa_{0_e}^+\}$$

$$\kappa_{n_c}\{\kappa_{n_c}^- \longrightarrow \kappa_{n_c}^+\}$$

$$\kappa_{n_e}\{\kappa_{n_e}^- \longrightarrow \kappa_{n_e}^+\} \qquad \kappa_{n_c}^- = 0_c$$

Since $\forall$ is the set of roots for κ_0, and given that $\forall_0$ equals κ_0, it follows that:

$$2^{\kappa_0} = 1 + \kappa_0$$

$$\updownarrow \qquad \Big\} \quad \begin{array}{l} \forall_0 = \kappa_0 \\ \forall_{0_c} = \kappa_{0_e} \\ \forall_{0_c}^{-} = \kappa_{0_e}^{+} \end{array}$$

$$\left.\begin{array}{l} 2^{\forall_{0_c}^{-}} = 1 + \forall_{0_c}^{-} \\ 2^{\forall_{0_c}^{+}} = 1 + \forall_{0_c}^{+} \end{array}\right\} \begin{array}{l} 2^{\forall_{0_c}} = 1 + \forall_{0_c} \\ \forall_{0_c}\{\forall_{0_c}^{-} \longrightarrow \forall_{0_c}^{+}\} \end{array}$$

$$\Big\} \quad \forall_{0_c}^{+} = \forall_{0_e}^{-} = \forall_0 \qquad \Bigg\} \quad \begin{array}{l} 2^{\forall_0} = 1 + \forall_0 \\ \forall_0\{\forall_{0_c} \longrightarrow \forall_{0_e}\} \end{array}$$

$$\left.\begin{array}{l} 2^{\forall_{0_e}^{-}} = 1 + \forall_{0_e}^{-} \\ 2^{\forall_{0_e}^{+}} = 1 + \forall_{0_e}^{+} \end{array}\right\} \begin{array}{l} 2^{\forall_{0_e}} = 1 + \forall_{0_e} \\ \forall_{0_e}\{\forall_{0_e}^{-} \longrightarrow \forall_{0_e}^{+}\} \end{array}$$

$$\vdots \qquad \Big\} \quad \forall_{0_e}^{\max +} \text{ and } \forall_{1_c}^{\min -}$$

$\forall$ can be used to account for the continuity between infinitesimal and finite numbers. It is the set of intermediate quantities that range between the most upper bound of the infinitesimals and that infinitesimal being raised to smaller and smaller roots.

Therefore, it becomes deducible that $\forall_0$, which equals $\kappa_{0,}$ when raise to an infinitesimal enough root equals one; given that smaller and smaller decimal numbers, when raised to smaller and smaller roots, yield finite numbers that are closer and closer to one.

$$\left.\begin{array}{l}
\left.\begin{array}{l} 2^{\forall_{0_c}} = 1+\forall_{0_c} \\ 2^{\forall_{0_e}} = 1+\forall_{0_e} \end{array}\right\} \begin{array}{l} 2^{\forall_0} = 1+\forall_0 \\ \forall_0\ \{\forall_{0_c} \longrightarrow \forall_{0_e}\} \end{array} \\[2ex]
\left.\begin{array}{l} 2^{\forall_{1_c}} = 1+\forall_{1_c} \\ 2^{\forall_{1_e}} = 1+\forall_{1_e} \end{array}\right\} \begin{array}{l} 2^{\forall_1} = 1+\forall_1 \\ \forall_1\ \{\forall_{1_c} \longrightarrow \forall_{1_e}\} \end{array} \\[2ex]
\left.\begin{array}{l} 2^{\forall_{2_c}} = 1+\forall_{2_c} \\ 2^{\forall_{2_e}} = 1+\forall_{2_e} \end{array}\right\} \begin{array}{l} 2^{\forall_2} = 1+\forall_2 \\ \forall_2\ \{\forall_{2_c} \longrightarrow \forall_{2_e}\} \end{array} \\[2ex]
\quad \downarrow \ \uparrow \qquad \} \ \text{super order} \\[2ex]
\left.\begin{array}{l} 2^{\forall_{n_c-2}} = 1+\forall_{n_c-2} \\ 2^{\forall_{n_e-2}} = 1+\forall_{n_e-2} \end{array}\right\} \begin{array}{l} 2^{\forall_{n-2}} = 1+\forall_{n-2} \\ \forall_{n-2}\ \{\forall_{n_c-2} \longrightarrow \forall_{n_e-2}\} \end{array} \\[2ex]
\left.\begin{array}{l} 2^{\forall_{n_c-1}} = 1+\forall_{n_c-1} \\ 2^{\forall_{n_e-1}} = 1+\forall_{n_e-1} \end{array}\right\} \begin{array}{l} 2^{\forall_{n-1}} = 1+\forall_{n-1} \\ \forall_{n-1}\ \{\forall_{n_c-1} \longrightarrow \forall_{n_e-1}\} \end{array}
\end{array}\right\} \forall\ \{\forall_0 \longrightarrow \forall_n\}$$

$$\vdots$$

$$\left.\begin{array}{l} 2^{\forall_{n_c}} = 1+\forall_{n_c} = 1+0.999\ldots \\ 2^{\forall_{n_e}} = 1+\forall_{n_e} = 1+1 \end{array}\right\} \begin{array}{l} 2^{\forall_n} = 1+\forall_n \\ \forall_n\ \{\forall_{n_c} \longrightarrow \forall_{n_e}\} \end{array}$$

Therefore, it can be shown that the set $\forall$, set of all roots of κ_0, ambulates over a range from the most upper bound infinitesimal κ_0 to one. Thus, κ_0 and one are both members of the set $\forall$.

$$\forall_0 = \kappa_0 \qquad\qquad \forall\ \{\forall_0 \longrightarrow \forall_n\} \qquad\qquad \forall\ \{\kappa_0 \longrightarrow 1\}$$

Since Ω is the set of roots for $\aleph_0$, and given that Ω_0 equals $\aleph_0$ and that Ω_n in its expanded form equals one, it follows that:

$$2^{\Omega^+_{n_e}} = 1 + 1 + 0 = 2$$

$$\left.\begin{array}{l} 2^1 = 1 + 1 = 2 \\ \updownarrow \end{array}\right\} \quad \Omega^+_{n_e} = 1$$

$$\Omega^+_{n_e} = \aleph_0^{\kappa^-_{n_c}} = 1 + \kappa^-_{n_c} = 1 + 0$$

$$\Omega^-_{n_c} = \aleph_0^{\kappa^+_{n_c}} = \aleph_0^{\kappa_{n_c}} = 1 + \kappa_{n_c}$$

$$\Omega_n = \aleph_0^{\kappa_0} = 1 + \kappa_0$$

$$\left.\begin{array}{l} 2^{\Omega^+_{n_e}} = 1 + \Omega^+_{n_e} \\ 2^{\Omega^-_{n_e}} = 1 + \Omega^-_{n_e} \end{array}\right\} \begin{array}{l} 2^{\Omega_{n_e}} = 1 + \Omega_{n_e} \\ \Omega_{n_e}\{\Omega^-_{n_e} \rightarrow \Omega^+_{n_e}\} \end{array}$$

$$\Big\} \quad \Omega^+_{n_c} = \Omega^-_{n_e} = \Omega_n \qquad \Bigg\} \begin{array}{l} 2^{\Omega_n} = 1 + \Omega_n \\ \Omega_n\{\Omega_{n_c} \rightarrow \Omega_{n_e}\} \end{array}$$

$$\left.\begin{array}{l} 2^{\Omega^+_{n_c}} = 1 + \Omega^+_{n_c} \\ 2^{\Omega^-_{n_c}} = 1 + \Omega^-_{n_c} \end{array}\right\} \begin{array}{l} 2^{\Omega_{n_c}} = 1 + \Omega_{n_c} \\ \Omega_{n_c}\{\Omega^-_{n_c} \rightarrow \Omega^+_{n_c}\} \end{array}$$

$$\vdots \quad \Big\} \quad \Omega^{\min -}_{n_c} \text{ and } \Omega^{\max +}_{n_e - 1}$$

Ω can be used to account for the continuity between finite numbers and the infinites or cardinal numbers. It is the set of intermediate quantities that range between the least upper bound infinite number and that infinite number being raised to deeper and deeper roots. It becomes deducible that $\aleph_0$ raise to an infinitesimal enough root equals one. Thus, Ω approached one from $\aleph_0$.

$$\left.\begin{aligned}2^{\Omega_{n_e}} &= 1+\Omega_{n_e} = 1+\sqrt[\aleph_{n_e}]{\aleph_0} = 1+\aleph_{0_c}^{\kappa_{n_c}} = 2+\kappa_{n_c}\\ 2^{\Omega_{n_c}} &= 1+\Omega_{n_c} = 1+\sqrt[\aleph_{n_c}]{\aleph_0} = 1+\aleph_{0_e}^{\kappa_{n_e}} = 2+\kappa_{n_e}\end{aligned}\right\} \begin{array}{l} 2^{\Omega_n} = 1+\Omega_n \\ \Omega_n\ \{\Omega_{n_c}\to\Omega_{n_e}\}\end{array}$$

$$\left.\begin{aligned}2^{\Omega_{n_e}-1} &= 1+\Omega_{n_e-1} = 1+\sqrt[\aleph_{n_e}-1]{\aleph_0} = 1+\aleph_{0_c}^{\kappa_{n_c}-1} = 2+\kappa_{n_c}-1\\ 2^{\Omega_{n_c}-1} &= 1+\Omega_{n_c-1} = 1+\sqrt[\aleph_{n_c}-1]{\aleph_0} = 1+\aleph_{0_e}^{\kappa_{n_e}-1} = 2+\kappa_{n_e}-1\end{aligned}\right\} \begin{array}{l} 2^{\Omega_{n_c}-1} = 1+\Omega_{n_c-1} \\ \Omega_{n-1}\{\Omega_{n_c-1}\to\Omega_{n_e-1}\}\end{array}$$

$$\left.\begin{aligned}2^{\Omega_{n_e}-2} &= 1+\Omega_{n_e-2} = 1+\sqrt[\aleph_{n_e}-2]{\aleph_0} = 1+\aleph_{0_c}^{\kappa_{n_c}-2} = 2+\kappa_{n_c}-2\\ 2^{\Omega_{n_c}-2} &= 1+\Omega_{n_c-2} = 1+\sqrt[\aleph_{n_c}-2]{\aleph_0} = 1+\aleph_{0_e}^{\kappa_{n_e}-2} = 2+\kappa_{n_e}-2\end{aligned}\right\} \begin{array}{l} 2^{\Omega_{n_c}-2} = 1+\Omega_{n_c-2} \\ \Omega_{n-2}\{\Omega_{n_c-2}\to\Omega_{n_e-2}\}\end{array}$$

$\downarrow\ \uparrow$ } super order

$$\left.\begin{array}{l} 2^{\Omega_2} = 1+\sqrt[3]{\aleph_0} = 1+\Omega_2 \ \}\ \Omega_2\{\Omega_{2_c}\to\Omega_{2_e}\} \\[2ex] 2^{\Omega_1} = 1+\sqrt[2]{\aleph_0} = 1+\Omega_1 \ \}\ \Omega_1\{\Omega_{1_c}\to\Omega_{1_e}\} \\[2ex] \Omega_0 = \aleph_0 \end{array}\right\} \Omega\{\Omega_0\to\Omega_n\}$$

It can be shown that the set Ω, set of all roots of $\aleph_0$, perambulates over a range from the least upper bound infinite $\aleph_0$ to one. Therefore, $\aleph_0$ and one are both members of the set Ω.

$$\Omega\ \{\Omega_0\to\Omega_n\} \qquad \Omega\ \{\aleph_0\to 1\}$$

Finally, it can be shown that the set $\aleph$, set of all infinites or cardinal numbers, perambulates as an ordered series over a range from the least upper bound infinite number $\aleph_0$ to the most upper bound infinite number $\aleph_n$. It becomes deducible that the expanded or trans-infinite form of zero is the most upper bound infinite number, it is the power set least the set of the continuum, and it is a member of the set $\aleph$.

$$\left.\begin{array}{rcl} & 2^{\aleph_{0_c}^-} = & \aleph_{0_e}^+ \\ \aleph_{0_c}^+ = & \aleph_{0_e}^- = & \aleph_0 \\ & 2^{\aleph_{0_e}^+} = & \aleph_{1_c}^- \end{array}\right\} \aleph_0 \ \{ \aleph_{0_c} \longrightarrow \aleph_{0_e} \}$$

$$\left.\begin{array}{rcl} & 2^{\aleph_{1_c}^-} = & \aleph_{1_e}^+ \\ \aleph_{1_c}^+ = & \aleph_{1_e}^- = & \aleph_1 \\ & 2^{\aleph_{1_e}^+} = & \aleph_{2_c}^- \end{array}\right\} \aleph_1 \ \{ \aleph_{1_c} \longrightarrow \aleph_{1_e} \}$$

$$\left.\begin{array}{rcl} & 2^{\aleph_{2_c}^-} = & \aleph_{2_e}^+ \\ \aleph_{2_c}^+ = & \aleph_{2_e}^- = & \aleph_2 \\ & 2^{\aleph_{2_e}^+} = & \aleph_{3_c}^- \end{array}\right\} \aleph_2 \ \{ \aleph_{2_c} \longrightarrow \aleph_{2_e} \}$$

$$\downarrow \ \uparrow \quad \} \ \text{super order}$$

$$\aleph \ \{ \aleph_0 \longrightarrow \aleph_n \}$$

$$\left.\begin{array}{rcl} & 2^{\aleph_{n_e-3}^+} = & \aleph_{n_c-2}^- \\ \aleph_{n_c-2}^+ = & \aleph_{n_e-2}^- = & \aleph_{n-2} \\ & 2^{\aleph_{n_c-2}^-} = & \aleph_{n_e-2}^+ \end{array}\right\} \aleph_{n-2} \ \{ \aleph_{n_c-2} \longrightarrow \aleph_{n_e-2} \}$$

$$\left.\begin{array}{rcl} & 2^{\aleph_{n_e-2}^+} = & \aleph_{n_c-1}^- \\ \aleph_{n_c-1}^+ = & \aleph_{n_e-1}^- = & \aleph_{n-1} \\ & 2^{\aleph_{n_c-1}^-} = & \aleph_{n_e-1}^+ \end{array}\right\} \aleph_{n-1} \ \{ \aleph_{n_c-1} \longrightarrow \aleph_{n_e-1} \}$$

$$\left.\begin{array}{rcl} & 2^{\aleph_{n_e-1}^+} = & \aleph_{n_c}^- \\ \aleph_{n_c}^+ = & \aleph_{n_e}^- = & \aleph_n \\ & 2^{\aleph_{n_c}^-} = & \aleph_{n_e}^+ \end{array}\right\} \aleph_n \ \{ \aleph_{n_c} \longrightarrow \aleph_{n_e} \}$$

$$\updownarrow \quad \} \quad \aleph_{n_e}^+ = 0_e$$

$$2^{0_e} = C$$

Since $\aleph_{n_e}^+$ is the greatest set least its power set C, and since $\aleph_{n_e}^+$ is the greatest subset of C, since C is the greatest set and has no power set, and since C is a power set, it follows that the power set of the greatest set least the set of the continuum is the continuum.

Therefore, zero ambulates between a compressed common form and an expanded infinite form. Zero as a whole, undivided, is the continuum.

$$0 \ \{ \ 0_c \ \rightarrow 0_e \ \}$$

The continuum in power set form ambulates between zero in its differentiated forms as the compressed form of the continuum (C_c) and zero as a whole in its undifferentiated form as the expanded form of the continuum (C_e).

$$C \ \{\{ 0_e \rightarrow 0_c\}, \{ 0_c \rightarrow 0_e \}\}$$

$$C \ \{\{0\}, \{ 0_c \rightarrow 0_e \}\}$$

$$C \ \{ \ C_e \rightarrow C_c \ \}$$

It has been shown that κ, the set of all infinitesimals, ambulates between κ_0, the least lower bound of the infinitesimals, and 0_c, the compressed or common form of zero.

$$\kappa \ \{ \ \kappa_0 \rightarrow 0_c \ \}$$

Since $\forall$ is the set of all of the roots of κ_0 and given that $\forall_0 = \kappa_0$,
$\forall$ ambulates between κ_0 and 1.

$$\forall \ \{ \ \kappa_0 \rightarrow 1 \ \}$$

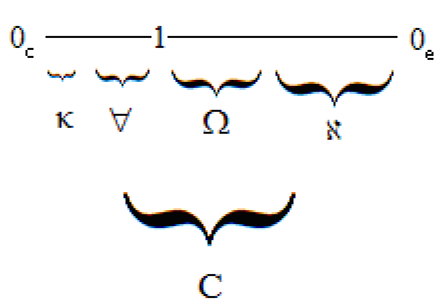
Since Ω is the set of all of the roots of $\aleph_0$ and given that $\Omega_0 = \aleph_0$,
Ω ambulates between $\aleph_0$ and 1.

$$\Omega \ \{ \ \aleph_0 \rightarrow 1 \ \}$$

It has been shown that $\aleph$, the set of all infinites or cardinal numbers, ambulates between $\aleph_0$, the least upper bound of the infinites, and 0_e, the expanded or infinite form of zero.

$$\aleph \ \{ \ \aleph_0 \rightarrow 0_e \ \}$$

Finally, the continuum C, as a power set of its final term, ambulates between the undifferentiated form (0) and the differentiated forms (0_c, 0_e) of zero. Zero, as a whole, is the continuum.



$$\left.\begin{array}{l} \kappa \ \{ \ \kappa_0 \rightarrow 0_c \ \} \\ \forall \ \{ \ \kappa_0 \rightarrow 1 \ \} \\ \Omega \ \{ \ \aleph_0 \rightarrow 1 \ \} \\ \aleph \ \{ \ \aleph_0 \rightarrow 0_e \ \} \end{array}\right\} C \ \{\{0\}, \{ 0_c \rightarrow 0_e \}\}$$

$$0 \ \{ \ 0_c \ \rightarrow 0_e \ \}$$

Cannonized Cardinal Continuum

$C \quad \longleftrightarrow$

$\kappa \qquad \forall \qquad 1 \qquad \Omega \qquad \aleph$

$\{0_c \longrightarrow 0_e\}$

$\{\kappa_n \leftarrow \kappa_0\} \quad 1 \quad \{\aleph_0 \rightarrow \aleph_n\}$

$\{\forall_0 \leftarrow \forall_n\}$

$\{\Omega_n \rightarrow \Omega_0\}$

1.16- Limits

We can see that the definitions found in calculus for either of the limits for $f(n) = 1/n$, where n approaches zero or where n approaches infinity, would have to be changed to account for these new treatments of infinites and infinitesimals.

Classical Calculus:

$\lim_{n\to\infty} f(n) = 1/n = 0$ $\qquad$ $\lim_{n\to 0} f(n) = 1/n = \infty$

$1 / \infty = 0$ $\qquad$ $1 / 0 = \infty$

There is justification for questioning these classical definitions, and replacing them with:

C3 Calculus:

$\lim_{n\to\aleph} f(n) = 1/n = \kappa$ $\qquad$ $\lim_{n\to\kappa} f(n) = 1/n = \aleph$ $\qquad$ $\lim_{n\to 0} f(n) = 1/n = 0$

$1 / \aleph = \kappa$ $\qquad$ $1 / \kappa = \aleph$ $\qquad$ $1 / 0 = 0$

The consistency of the incompleteness completes the inconsistency of the continuum verses the inconsistency of the completeness incompletes the consistency of the continuum.

There is an obvious channel of pattern supporting C3 which is inherent to the interplay of transfinite numbers through special algebraic rules such that communicable equations can be derived to maintain the consistency of the structure of the argument for C3.

2.0- C3 and the Conventional Approach

The questions facing the C3 argument are:

1. Is the sequence $\{0_c, \kappa, \forall, 1, \Omega, \aleph, 0_e\}$ really accountable for the continuum?
2. Is $\{0_c, \kappa, \forall, 1, \Omega, \aleph, 0_e\}$ a masked sequence of the set of natural or real numbers?
3. Are the C3 algebraic treatments of zero, infinitesimals, and infinites sound?
4. Is C3 compatible with and/or resolve issues in Non-Standard Analysis?

C3 reasons well reworking the assumption:

$$2^{\aleph_0} = \aleph_1 \quad \text{into} \quad 2^{\aleph_{0_e}^{+}} = \aleph_{1_c}^{-} \qquad \text{and} \qquad \aleph_1 \neq C$$

The cardinal number $\aleph_1$ is not finite nor does it have the same cardinality as $\aleph_0$, and $\aleph_1$ has cardinality strictly greater than $\aleph_0$, however, there is another way of treating $\aleph_0$ and $\aleph_1$ that reveals more about the continuum and the accountability of numbers.

C3 is congruent with the assumption that the set of real numbers, **R**, is $2^{\aleph_0} = \aleph_1$, however ambulation opens another dimension to the accountability of infinite sets such as **R** and allows for an infinitude of infinites greater than **R** to be counted. Right away, ambulation diversifies the set **R** into **R'** for more specific accountability:

Real numbers	Ambulating Real numbers
$\mathbf{R} = 2^{\aleph_0} = \aleph_1 = \{0, \forall, \Omega\}$	$\mathbf{R}' = 2^{\aleph_{0_e}^{+}} = \aleph_{1_c}^{-} = \{0, \forall, \Omega\}$
where	where
$\Omega_0 \neq \aleph_0 \quad \Omega_0 < \aleph_0$	$\Omega_0 = \aleph_0$
$\forall_0 \neq \kappa_0 \quad \forall_0 > \kappa_0$	$\forall_0 = \kappa_0$

A more uncountable set **R***, the set of all hyper-real and transfinite numbers, has the cardinality of $\aleph_n$. **R*** < C because as a set, **R*** does not ambulate and does not contain the differentiated forms of zero as limits. Ambulation can diversify **R*** into **R**** where:

Hyper-real/tranfinite numbers	Ambulating Hyper-real/transfinite numbers
$\mathbf{R}^* = 2^{\aleph_{n-1}} = \aleph_n = \{0, \kappa, \forall, 1, \Omega, \aleph\}$	$\mathbf{R}^{**} = 2^{\aleph_{n_c}^{-}} = \aleph_{n_e}^{+} = 0_e = \{0, \kappa, \forall, 1, \Omega, \aleph\}$
where	where
$\aleph_n \neq C \quad \aleph_n < C$	$\aleph_{n_e}^{+} = 0_e$
$\kappa_n \neq 0 \quad \kappa_n > 0$	$\kappa_{n_c}^{-} = 0_c$

The concept of ordinals is accounted for by adopting the use of ambulation with transfinite numbers. The use of ordinals as position is implicitly compatible to C3. The uncountable set ω_1, the set of all countable ordinal numbers, has a cardinality of $\aleph_1$. Since it can be shown using the axiom of choice that it is the smallest uncountable cardinal number ($\aleph_0$ being the last set of 'countable' numbers). The idea of a cardinal continuum ($\aleph_0, \aleph_1, \aleph_2, \ldots, \aleph_n$), or infinite degrees of infinity, is deducible. Since it is arguable, using C3, that though the cardinality of **R** is $\aleph_1$ and though $\aleph_1$ is uncountable, $\aleph_1 \neq C$, and the cardinality of the continuum should be shown to be significantly larger than the cardinality of **R**. According to C3, the cardinality of the continuum C is the power set of the last term 0_e, which is infinitely greater than **R**. Using a reverse standard parts method, finite numbers can be shown to be ordinals. C3 accounts for numeration from $\kappa_n, \kappa_{n-1}, \kappa_{n-1}, \ldots, \kappa_2, \kappa_1, \kappa_{0, \ldots}$, 1, 2, 3, … , $\aleph_0, \aleph_1, \aleph_2, \ldots, \aleph_n$ etc. Since ω is the ordinal associated with the set of natural numbers $\aleph_0$, and since ω_1 is associated with $\aleph_1$, the compressed forms of $\aleph_0$ can be associated with the ordinals ω, $\omega+1$, $\omega+2$, … $(\omega+\omega)$, $(\omega+\omega+\omega)$, …, $(\omega\cdot\omega)$, $(\omega\cdot\omega\cdot\omega)$, etc. and the expanded forms of the cardinal number $\aleph_0$ can be associated with the ordinals ω^ω, ε_0, etc., and ω_1 the set of all countable ordinals, can be accounted for in C3 as $\aleph_1$. The sequence in C3 of $\aleph_1, \aleph_2, \ldots, \aleph_n$ accounts for the uncountable ordinals ω_1, ω_2, ω_3, etc. and the set of all uncountable ordinals can be expressed through the relationship of $\aleph_n$ to 0_e.

(see Appendix)

$$2^{0_e} = C$$

2.1- Antithetical Proof

0_e is the set of all numbers, C is the power set of 0_e, and C contains a member (zero) that, when in set form, accounts for members not found anywhere within the set 0_e and are not numbers in any ordinary sense, namely the un-ordinals.

It can easily be deduced that 0 in the un-differentiated form is an un-ordinal, given that treating zero algebraically can derive any value.

$0^0 = 0^{(1-1)}$	$0^0 = 0/1 \text{ x } 1/0 = 0/0 = 0$
$0^0 = 0^{(1)} \text{ x } 0^{(-1)}$	$0^0 = 0/1 \text{ x } 1/0 = 0/0 = 1$
$0^0 = 0/1 \text{ x } 1/0$	$0^0 = 0/1 \text{ x } 1/0 = 0/0 = 2$
$0^0 = 0/1 \text{ x } 1/0 = 0/0$	$0^0 = 0/1 \text{ x } 1/0 = 0/0 = 3$
$0^0 = 0/1 \text{ x } 1/0 = 0/0 = 1$	$0^0 = 0/1 \text{ x } 1/0 = 0/0 = n$

If we recall, there is a continua of un-ordinals that exists for every ordinal. We can see this by the fact that for every correct equality, there is a continua of incorrect equalities.

Thus, the set of all ordinals and un-ordinals, namely C, would look something like this:

$$\begin{bmatrix} 0x0=0 & \ldots & 1x1=0 & \ldots & 1x2=0 & \ldots & 1x3=0 & \ldots\ldots\ldots & 1xn=0 \\ 0x1=0 & \ldots & 1x1=1 & \ldots & 1x2=1 & \ldots & 1x3=1 & \ldots\ldots\ldots & 1xn=1 \\ 0x2=0 & \ldots & 1x1=2 & \ldots & 1x2=2 & \ldots & 1x3=2 & \ldots\ldots\ldots & 1xn=2 \\ 0x3=0 & \ldots & 1x1=3 & \ldots & 1x2=3 & \ldots & 1x3=3 & \ldots\ldots\ldots & 1xn=3 \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ 0xn=0 & \ldots & 1x1=n & \ldots & 1x2=n & \ldots & 1x3=n & \ldots\ldots\ldots & 1xn=n \end{bmatrix}$$

What becomes apparent is that even though the set of un-ordinals is uncountable, at first, it appears to possibly be greater than the set of ordinals. A Cantor diagonal style proof can be used to show that both sets are uncountable and have the same cardinality or alternatively that the set of all un-ordinals has a cardinality less than that of the set of all ordinals. Either way, the cardinality of the powerset of the set of all ordinals is the continuum C. Therefore, within C3, no set larger than C has been conceived where in addition C is also a member thereof (set of all sets as a member of itself).

(see Appendix)

$$\begin{bmatrix} 0x0=0 & \ldots & 1x1=0 & \ldots & 1x2=0 & \ldots & 1x3=0 & \ldots\ldots\ldots & 1xn=0 \\ 0x1=0 & \ldots & 1x1=1 & \ldots & 1x2=1 & \ldots & 1x3=1 & \ldots\ldots\ldots & 1xn=1 \\ 0x2=0 & \ldots & 1x1=2 & \ldots & 1x2=2 & \ldots & 1x3=2 & \ldots\ldots\ldots & 1xn=2 \\ 0x3=0 & \ldots & 1x1=3 & \ldots & 1x2=3 & \ldots & 1x3=3 & \ldots\ldots\ldots & 1xn=3 \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ 0xn=0 & \ldots & 1x1=n & \ldots & 1x2=n & \ldots & 1x3=n & \ldots\ldots\ldots & 1xn=n \end{bmatrix}$$

If we take the complete product of the set of all ordinals by multiply all members of the set of all ordinals together with their reciprocal members, we get a complete product of 1. The reason is that for every member of the set of all ordinals there exists a reciprocal member such that when they are all multiplied together, the complete product is 1.

$$\left[\frac{1}{n} \cdot \ldots \cdot \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot n \right]$$

Therefore, the set of all ordinals has a cardinality of 0_e and a complete product of 1.

If we consider the set of all un-ordinals, we find that though a continua of un-ordinals exists for every ordinal, the complete product of each continua is the reciprocal value of the ordinal it extends from. Therefore, it would be the case that the complete product of the set of all un-ordinals is also 1. From this we can deduce that both the set of all ordinals and the set of all un-ordinals have the same cardinality, namely 0_e. However, it turns out that something has not been considered and has been omitted from both sets; that being zero in the undifferentiated form.

Let zero differentiated as reciprocal compressed and expanded forms existing at both extremes of the continuum exist within the set of all ordinals and let zero undifferentiated without a reciprocal, as its own reciprocal, exist within the non-set of all un-ordinals. Clearly, if there is a candidate for the non-set of un-ordinals zero would be it. Undifferentiated zero is an odd un-ordinal because it has no reciprocal. It becomes even ordinals when its compressed and expanded forms are differentiated into a reciprocal relationship.

Undifferentiated: $1/0 = 0$ *Differentiated*: $1/0_e = 0_c$ $1/0_c = 0_e$

Therefore, since all the un-ordinals have a reciprocal other than zero, the complete product of the non-set of all un-ordinals, with the undifferentiated zero as a member, is zero. So the difference between the cardinality of the set of ordinals and the set of un-ordinals is zero; in other words, nothing. Inadvertently, this also can be used to show that there is no greater cardinality than C.

Agains, since un-ordinals are meaningless outside the use of un-ordinals to establish the continuum as having the greatest possible cardinality, we have neither conceived of a set of all sets that contains itself as a member nor have we conceived a cardinality greater than C.

Zero perambulates the continuum in two ways:

1. Zero, whose product is any point <u>or</u> interval along the continuum, exists at both extreme ends of the continuum.

2. Zero is the complete product of all ordinals and un-ordinals.

The continuum is the full perambulation of zero by its proliferation of number. Proliferating away from zero are the very numbers that, when culminated together in the form of a continuum, consolidate into the value of zero. Zero comes together and makes number, and numbers comes together and makes zero.

There are three forms of zero:

0_c - Compressed or common form- approaching zero from the infinitesimal side

0_e - Expanded or infinite form- approaching zero from the infinite side

0 - Complex Product (super continuity)- undifferentiated zero.

0_e is the set of all ordinals, C is the power set of 0_e and C is the set of all ordinals and the non-set of all un-ordinals.

$\aleph_0$ {1,2,3,4…}- the natural numbers $\aleph_0$; the initial infinite (cardinal) number, least upper bound of the infinites. None of the members of this set ambulate though the set as a whole does.

R $\{0, \forall, \Omega\}$- the real numbers **R** (or $\aleph_1$), where **R** ranges from $\forall_0$ to $\forall_n$ and Ω_0 to Ω_n;
$\forall_n = 1,\ \Omega_n = 1;\quad \forall_0 \neq \kappa_0,\ \Omega_0 \neq \aleph_0;\quad \forall_0 > \kappa_0,\ \Omega_0 < \aleph_0$

R’ $\{0, \forall, \Omega\}$- the ambulating real numbers **R’** ranging from $\forall_0$ to $\forall_n$ and Ω_0 to Ω_n;
$\forall_n = 1,\ \Omega_n = 1;\quad \forall_0 = \kappa_0,\ \Omega_0 = \aleph_0;\quad \forall_0 \geq \kappa_0,\ \Omega_0 \leq \aleph_0$

R*$\{0, \kappa, \forall, 1, \Omega, \aleph\}$- the hyper-real numbers **R***, ranging from κ_0 to κ_n and $\aleph_0$ to $\aleph_n$;
$0 \neq \kappa_n,\ C \neq \aleph_n;\quad 0 < \kappa_n < \kappa_0;\quad \aleph_0 < \aleph_n < C$

R**$\{0, \kappa, \forall, 1, \Omega, \aleph\}$- the ambulating hyper-real numbers **R****,
ranging from κ_0 to κ_n and $\aleph_0$ to $\aleph_n$;
$\kappa^-_{n_c} = 0_c,\quad \aleph^+_{n_e} = 0_e$

0_D $\{0_c \rightarrow 0_e\}$- zero in the differentiated form is the range of all numbers ambulating the entire continuum ranging from 0_c to 0_e.

0_U $\{0_e \rightarrow 0_c\}$- zero in the undifferentiated form is the range of all un-ordinals excluded from the continuum with zero range.

C $\{\{0_e \rightarrow 0_c\}, \{0_c \rightarrow 0_e\}\}$- the continuum in the power set form is the differentiated and undifferentiated forms of zero.
C $\{\{0\}, \{0_c \rightarrow 0_e\}\}$

C $\{C_e \rightarrow C_c\}$- the compressed form of the continuum C_c is $0_c \rightarrow 0_e$ and the expanded form of the continuum C_e is $0_e \rightarrow 0_c$ (since from {0} all numbers extend).

C [0,1]- the continuum has the complete product of zero for the set of un-ordinals and a complex product of one for the set of ordinals.

C [0]- the continuum has a complete product of zero.

2.2- Compatibility of C3 with Non-Standard Analysis

C3 is compatible with Non-Standard Analysis in very specific ways. The C3 treatment of infinitesimals and infinites accounts for hyper-reals consistently with the extension principle, the transfer principle, the real statements, and L'Hopital's Rule with very specific exceptions. C3 sets a cardinal continuum of infinites and infinitesimals in corresponding reciprocal and algebraic relationships leading to a definition of 1/0 then allowing for the determination of the Indeterminate Forms. These more complete definitions of the role of infinites, infinitesimals, and zero clarify the "ghosts of departed quantities" issues that surround Δx and dx in Differentiation and Integration in Calculus by modifying the Standard Parts Method. <u>The utility of C3 extends beyond the Continuum Hypothesis to Infinitesimal Calculus and Non-Standard Analysis seamlessly</u>.

2.3- Standard Parts Method in Differentiation

The Standard Parts Method appears to be a slide-of-hand method.

Solving for $f'(x)$, where $y = x^3$, we find:

$$f'(x) = y + \Delta y = (x + \Delta x)^3$$

$$= y + \Delta y = (x +)(x + \Delta x)(x + \Delta x)$$

$$= y + \Delta y = (x + \Delta x)(x^2 + 2x\,\Delta x + \Delta x^2)$$

$$= y + \Delta y = x^3 + 2x^2\Delta x + x\,\Delta x^2 + x^2\Delta x + 2x\Delta x^2 + \Delta x^3$$

$$= \Delta y = -x^3 + x^3 + 2x^2\Delta x + x\,\Delta x^2 + x^2\Delta x + 2x\Delta x^2 + \Delta x^3$$

$$= \Delta y = -x^3 + x^3 + 2x^2\Delta x + x\,\Delta x^2 + x^2\Delta x + 2x\Delta x^2 + \Delta x^3$$

$$= \frac{\Delta y}{\Delta x} = \frac{3x^2\Delta x + x\,\Delta x^2 + 2x\Delta x^2 + \Delta x^3}{\Delta x}$$

$$= \frac{\Delta y}{\Delta x} = 3x^2 + 3x\Delta x + \Delta x^2$$

Using the Standard Parts Method, we would:

$$= \mathrm{st}\left(\frac{\Delta y}{\Delta x}\right) = \mathrm{st}(3x^2) + \mathrm{st}(3x\Delta x) + \mathrm{st}(\Delta x^2)$$

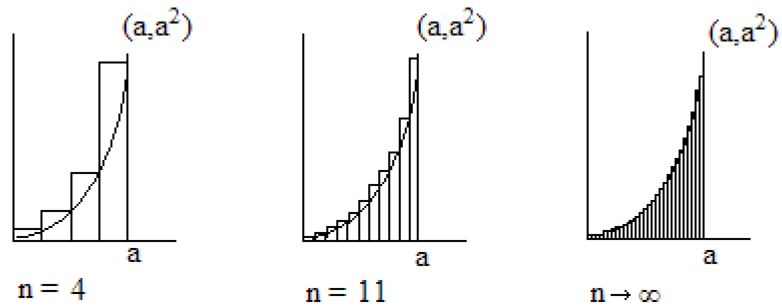
$$= \quad 3x^2 \quad + \quad 0 \quad + \quad 0$$

$$= 3x^2$$

Taking the Standard Part of a finite hyper-real number such as 3xΔx, where 3xΔx is an infinitesimal and where the Standard Part of an infinitesimal is 0, does not address the issue of how we transition from an infinitesimal increment Δx to 0 and how this is done without having to define 1/0 or allow for Δx = 0. In this way, the Standard Parts method is more like a selective reasoning or slide-of-hand, moving right along, using an ignore-the-issue-at-hand type method. "*What is* Δx *and how do we reduce it to zero without violating fundamental principles of arithmetic?*"- is the question that is not being addressed by Non-Standard Analysis in its application of the Standard Parts Method with Differentiation.

2.4- Standard Parts Method in Integration

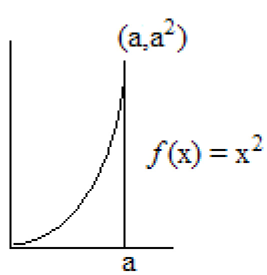
In integration, a series of rectangles are used to approximate the area under a curve. The width of these rectangles is reduced to smaller and smaller increments until they are infinitesimally wide. Then an infinites series of infinitesimally wide rectangles are summed together to give an exact value of area under the curve.



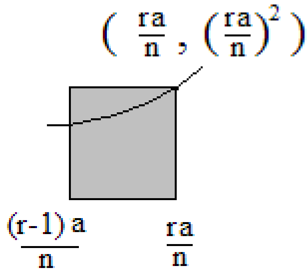
Two different sets of inequalities are used, an over estimate and an under estimate of A, that, as the widths of the rectangles tends towards zero and the number of them tends towards infinity. Then at the limit, the over and under estimate tend towards equaling the same value; namely A.

Example:

Find the area from x = 0 to x = a for the curve y = x^2.



Let a/n equal the width of n equally thin rectangles under the curve f(x). For some intermediate rectangle r, as an over estimate:



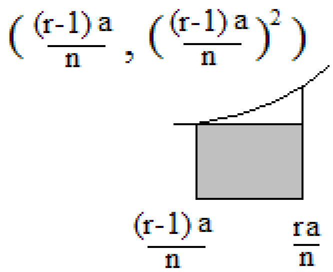
$\frac{ra}{n}$ x-coordinate

$\left(\frac{ra}{n}\right)^2$ y-coordinate

The area of rectangle r equals the base times the height relative to the highest point of rectangle r along *f*(x), which is:

$$A = x \cdot y = \frac{ra}{n} \times \left(\frac{ra}{n}\right)^2 = \frac{a^3}{n^3}\, r^2$$

As an underestimate:



$\frac{(r-1)\,a}{n}$ x-coordinate

$\left(\frac{(r-1)\,a}{n}\right)^2$ y-coordinate

The area of rectangle r equals the base times the height relative to the lowest point of rectangle r along *f*(x), which is:

$$A = x \cdot y = \frac{(r-1)\,a}{n} \times \left(\frac{(r-1)\,a}{n}\right)^2 = \frac{a^3}{n^3}\,(r-1)^2$$

Setting these two different estimates for the area of rectangle r as inequalities in a single equation, and using a series of rectangles to approximate the area under a curve, as the number of rectangle in the series approaches infinity the width of the rectangles tends towards zero trapping the value for the area between to equal limits.

$$\frac{a^3}{n^3}\sum_{r=1}^{n}(r-1)^2 < A < \frac{a^3}{n^3}\sum_{r=1}^{n} r^2$$

Solving the summations of the two series for the over and under estimates, we find:

$$\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{(n+1)(2n+1)}{6}$$

$$\sum_{r=1}^{n} (r\text{-}1)^2 = 0^2 + 1^2 + 2^2 + \ldots + (n-1)^2$$

Substituting (n-1) from the second series for n from the first series, we get:

$$\sum_{r=1}^{n} (r\text{-}1)^2 = \frac{(n-1)[\,(n-1)+1\,][\,2(n-1)+1\,]}{6}$$

$$= \frac{(n-1)\,n(2n-1)}{6}$$

Substituting into the inequalities we get:

$$\frac{a^3}{n^3} \times \frac{(n-1)\,n(2n-1)}{6} < A < \frac{a^3}{n^3} \times \frac{(n+1)(2n+1)}{6}$$

$$\frac{a^3}{6} \times \left(2 - \frac{3}{n} + \frac{1}{n^2} \right) < A < \frac{a^3}{6} \times \left(2 + \frac{3}{n} + \frac{1}{n^2} \right)$$

As n increases, the width of the rectangles gets smaller and as n approaches infinity, the widths tend towards zero. Using the Standard Parts method, we get:

$$\frac{a^3}{6} \times (\,2 - 0 + 0\,) < A < \frac{a^3}{6} \times (\,2 + 0 + 0\,)$$

$$\mathrm{st}\left(\frac{a^3}{6}\right) \times \mathrm{st}(\,2 - 0 + 0\,) < A < \mathrm{st}\left(\frac{a^3}{6}\right) \times \mathrm{st}(\,2 + 0 + 0\,)$$

$$\frac{a^3}{3} < A < \frac{a^3}{3}$$

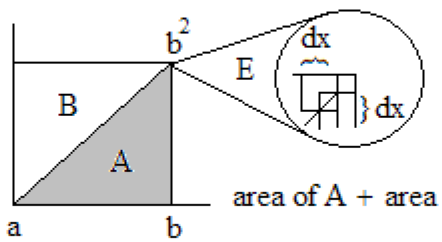
$$A = \frac{a^3}{3}$$

This is consistent, showing that the definite integral is equivalent with the anti-derivative, however, it again gives no reasoning for how this is done without defining 1/0.

2.5- Failure of the Standard Parts Method

Not only does the Standard Parts Method not explain how it avoids defining 1/0 though inadvertently using it, in this next example, it actually fails in defining the definite integral when looked at properly. Something else is going on within the Standard Parts method that the current definition of the process doesn't account for but is somehow dealt with in its underlying application.

Given b>a and b>0, evalutate the integral : $\int_0^b x dx$



area of A + area of B + area of E = b^2

Let a = 0

area of A = area of B = $\sum_0^b x dx$

where $\sum_0^b x dx$ is the summation

of the partitions:

$x_0 = a + \Delta x$; $x_1 = a + 2\Delta x$; $x_2 = a + 3\Delta x$; ... ; $x_n = a + n\Delta x$; b

of the sub-intervals:

$[x_0, x_1]$ $[x_1, x_2]$ $[x_2, x_3]$ ... $[x_n, b]$

The upper region B symmetrically has the same area as A. The Riemann sum, being a sum of a series of infinitesimally thin rectangles, has an infinitesimally small error with area E. Hence, the area of A plus the area of B plus the area of E = b^2. Each partition of E has a base of dx and a height of dx where the last term may be smaller than dx.

$$0 \leq \text{area of E} \leq bdx$$

Solving for A:

$$\text{area of A} + \text{area of B} < b^2 < \text{area of A} + \text{area of B} + \text{area of E}$$

$$2\sum_0^b x dx < b^2 < \left(2\sum_0^b x dx\right) + bdx$$

Since dx tends towards zero, bdx tends towards zero too, and in this way, E, or the error, is eliminated.

$$b^2 < \left(2\sum_0^b xdx\right) + 0$$

However, this comes at a price: what about xdx? Doesn't it zero out too? It could be argued that xdx is different from bdx given that the "x" in xdx is a dummy variable used representing the summation of a series of "x's", but this only drives the point further.

$$2\sum_0^b xdx \approx b^2$$

$$\sum_0^b xdx \approx \frac{b^2}{2}$$

Since bdx is infinitesimal, if dx is the width of each rectangle, and dx goes to zero, then not only does the width of each rectangle goes to zero but so does the integral area.

$$\int_0^b xdx = \int_p^p xdx = 0$$

where p is some arbitry
point between 0 and b.

Something has to be happening in the 'inner workings' of the Standard Parts method in order to yield the correct answer:

$$\int_0^b xdx = \frac{b^2}{2}$$

In examining these subtle problems with using the Standard Parts method in order to resolve the issues of the infinitesimals Δx and dx in Differentiation and Integration, C3 offers a comprehensive solution to the problem at hand that preserves the usability of the Standard Parts method and extends its utility into determining the Indeterminate Forms.

2.6- Determining the Indeterminate Forms

Indeterminate Forms exhibit ambulation. At first glace, some indeterminate forms lead to conclusions such as 0/0, however, some can be evaluated by an alternative method using L'Hôpital's rule and give definite answers. This yielding of a range of answers is exactly what we find in C3 as ambulation. Other Indeterminate Forms such as ∞/∞, 0^0, ∞^0, 1^∞, $\infty - \infty$, and $0\cdot\infty$, through the use of ambulation, can also be transformed into ambulating determinate forms. Consequentially and as will be demonstrated, the Standard Parts method and ambulation turn out to be two aspects of the same approach. Where taking the standard parts allows for treating hyperreals algebraically, ambulation allows for the precise definition of hyperreals allowing C3 to use them algebraically. Therefore, a closer examination at the utility of ambulation in C3 for determining the indeterminate forms will shed light on the C3 interpretation of the Standard Part method.

For the indeterminate form 0/0, let the differentiated forms of zero be used to determine the range of values for 0/0.

$$0_e \times 0_c = 1 \qquad \frac{1}{0_e} = 0_c \qquad \frac{1}{0_c} = 0_e$$

$$\frac{0_e}{0_e} = 1 \qquad \frac{0_c}{0_c} = 1 \qquad \frac{0_c}{0_e} < 1 \qquad \frac{0_e}{0_c} > 1$$

$$0_c \leq \frac{0_c}{0_e} < 1 < \frac{0_e}{0_c} \leq 0_e$$

Therefore, depending on the differentiated forms of zero, 0/0 has the continuum as a range of answers with the exception of 0/0 in the undifferentiated form of zero, which has only one answer: 0.

$$0 = \frac{0}{0} = \frac{1}{0} = 0$$

For the indeterminate form ∞/∞, let ∞ equal ℵ, for any given ℵ. Hence, for any ℵ/ℵ, so long as the ℵ in the numerator and the ℵ in the denominator are equal, ℵ/ℵ = 1.

$$\frac{\aleph_0}{\aleph_0} = \frac{\aleph_1}{\aleph_1} = \frac{\aleph_2}{\aleph_2} = \ldots = \frac{\aleph_n}{\aleph_n} = 1$$

Also, when ℵ/ℵ < 1 or ℵ/ℵ > 1, corresponding values can be determined (*please see definitions of the reciprocal relationships between infinites and infinitesimals*) such as:

$$\frac{\aleph_0}{\aleph_1} = \kappa_0 \qquad \text{since} \qquad \aleph_0 \times \kappa_1 = \kappa_0$$

For the indeterminate form 0^0, let the differentiated forms of zero be used to determine the range of values for 0^0.

$$0_c^{0_c} = 0_c \qquad 0_c^{0_e} = 0_c$$

$$0_e^{0_c} = 1 + 0_c \qquad 0_e^{0_e} = C$$

Using these we can derive:

$0^0 = 0 \qquad 0_c^{0_c} = 0_c$

$0^\infty = 0 \qquad 0_c^{0_e} = 0_c$

$\infty^0 = 1 \qquad 0_e^{0_c} = 1 + 0_c$

(see Appendix)

For the indeterminate form 1^∞, since $1^0 = 1$ and since zero exists at both ends of the continuum, i.e. zero is the continuum, $1^\infty = 1$.

$$1^{0_c} = 1^1 = 1^2 = 1^3 = \dots = 1^n = 1^{0_e}$$

From this we get:

$1^0 = 1 \qquad 1^{0_c} = 1$

$1^\infty = 1 \qquad 1^{0_e} = 1$

The indeterminate forms $\infty - \infty$ and $0 \cdot \infty$ are resolved by previously mentioned references to the reciprocal relationships and algebraic treatments of infinites and infinitesimals in C3 through the use of ambulation transforming them into determinate forms.

2.7- C3 and Differentiation

Returning to the issue regarding taking the standard parts as the final step in Differentiation, we find:

$$= \operatorname{st}\left(\frac{\Delta y}{\Delta x}\right) = \operatorname{st}(3x^2) + \operatorname{st}(3x\Delta x) + \operatorname{st}(\Delta x^2)$$

$$= \quad 3x^2 \quad + \quad 0 \quad + \quad 0$$

$$= 3x^2$$

If we let $\Delta x = \kappa_n$, and given that $\kappa_n = \varepsilon$, which is the initial increment and since κ_n subambulates to equal 0_c, $3x\Delta x$ and Δx^2 can be reduced to zero without any mystery and $\Delta x = 0_c$ can happen without violating the rules of arithmetic with C3 cleaning up the definition of the Standard Parts method. Let:

$$\operatorname{st}\left(\frac{\Delta y}{\Delta x}\right) = \operatorname{st}(3x^2) + \operatorname{st}(3x\Delta x) + \operatorname{st}(\Delta x^2)$$

$$\lim_{\Delta x \to \kappa_n} \frac{\Delta y}{\Delta x} = 3x^2 + 3x\,\kappa_n + \kappa_n^2 \qquad \Delta x = \kappa_n = \varepsilon$$

The total change in x, or the total subambulation of κ_n, equals the sum of the final change in x and the initial change in x.

Total change in x = *Final change in* x + *Initial change in* x

$x_t = x_f + x_i$

Since the final change in x subambulates to zero, $x_f = -\varepsilon$.

$$\lim_{\kappa_n \to 0_c} \frac{\Delta y}{\kappa_n} = 3x^2 + 3x(\kappa_n \to 0_c) + (\kappa_n \to 0_c)^2$$

$$= 3x^2 + 3x(\kappa_n \to (x_f + x_i)) + (\kappa_n \to (x_f + x_i))^2$$

$$= 3x^2 + 3x(\kappa_n \to (-\varepsilon + \varepsilon)) + (\kappa_n \to (-\varepsilon + \varepsilon))^2$$

$$= 3x^2 + 3x(\ 0_c\ ) + (\ 0_c\ )^2$$

$$= 3x^2$$

Please notice that in the C3 approach to Differentiation, it is the "subambulation" of the infinitesimal that reduces Δx to zero, not just some arbitrary wave of the Standard Parts hand.

The C3 approach to differentiation applies subambulation to the infinitesimal Δx taking into consideration the differentiated form of zero 0_c.

2.8- C3 and Integration

When applying the limit to the infinite Riemann of the series of partitions of sub-intervals as a function of x, we end up with an infinite summation of infinitesimal increments.

$$x_0 = a + \Delta x; \quad x_1 = a + 2\Delta x; \quad x_2 = a + 3\Delta x; \ \dots ; \quad x_n = a + n\Delta x; \ b$$

$$f(x_0),\ f(x_1),\ f(x_2),\ \dots,\ f(x_{n-1}) \ + f(x_n)(b - x_n)$$

$$\text{area of A} = \sum_{0}^{b} x dx$$

Referring back to the C3 treatment of the binomial $(1 + \kappa_0)^{\aleph_0}$, which yields an infinite summation of infinitesimal increments that perambulates over a range, specifically when treated as applied in **1.6 - 1.8**:

compressed additions of κ_n

$$(\kappa_n + \kappa_n + \kappa_n + ... + \kappa_n) = \kappa_{n_e}$$

expanded additions of κ_n

$$(\kappa_n + \kappa_n + \kappa_n + ... + \kappa_n) = (\aleph_n \times \kappa_n) = 1$$

The expanded additions of κ_n equals one, where the number of partitions is inversely proportional to the width of the partitions, and no error remains. Now we have a means of legitimately eliminating area E without losing xdx in the process.

So for bdx:

$$2\sum_0^b xdx < b^2 < \left(2\sum_0^b xdx\right) + b\,dx$$

$$\lim_{dx \to \kappa_n} 2\sum_0^b xdx < b^2 < \left(2\sum_0^b xdx\right) + b\,\kappa_n$$

$$\lim_{\kappa_n \to 0_c} 2\sum_0^b xdx < b^2 < \left(2\sum_0^b xdx\right) + b\,(\kappa_n \to 0_c)$$

$$+ b\,(\kappa_n \to (x_f + x_i))$$

$$+ b\,(\kappa_n \to (-\varepsilon + \varepsilon))$$

$$+ b\,(0_c)$$

$$\lim_{\kappa_n \to 0_c} 2\sum_0^b xdx < b^2 < \left(2\sum_0^b xdx\right) + 0$$

Now, using perambulation to account for the Standard Parts method in deriving the definite integral for the infinite Riemann sum, we find:

$$2\sum_0^b xdx \approx b^2$$

$$\lim_{dx \to \kappa_n} \sum_0^b xdx \approx \frac{b^2}{2}$$

Since dx is a dummy variable like x in this case and since they both are place holders for the summation of the partitions from 0 to b, given that the partitions are infinitesimally thin approaching zero, taking into account the infinite number of partitions that are proportional to the width of the partitions, as the infinitesimal width of a partition equals 0_c the infinite number of partitions equals 0_e.

$$\lim_{dx \to 1} \sum_{0}^{b} x\,(\aleph_n \times \kappa_n) \approx \frac{b^2}{2}$$

$$\lim_{dx \to 1} \sum_{0}^{b} x\,(0_c \times 0_e) \approx \frac{b^2}{2}$$

$$\lim_{dx \to 1} \sum_{0}^{b} x(1) \approx \frac{b^2}{2}$$

$$\int_0^b x\,dx = \frac{b^2}{2}$$

Since $0_c \cdot 0_e = 1$, the summation of dx, which is the expanded additions of κ_n that perambulates to equal one, preserves the function of x defining the definite integral.

Please notice that in the C3 approach to Integration, it is the "perambulation" of the infinitesimal that increases the 'dummy' dx status from zero to one.

Therefore, the C3 approach to Integration first applies subambulation to the dx times the error, and second perambulation to the infinitesimal dx taking into consideration the differentiated forms of zero, 0_c and 0_e.

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